Mathemat.l vol 4.

THE

SECTOR

AND

Plain Scale,

COMPARED.

CONTAINING

I. The Description of all the Lines upon the Sector and plain Scales.

II. The true use of the Sector made plain and easie in several Geometrical Problems, and in all the Cases of right lin'd Trigonometry.

III. All the preceeding Geometrical Problems, and Cases of right lin'd Trigonometry compared by the plain Scale, and proved by Mr. Gunter's Scale.

IV. All the preceeding Cases of right lin'd Trigonometry, performed Arithmetically, without the Help of any sort of Tables.

To which is annexed,

So much of Decimal Arithmatick, and the Extraction of the square Root, as is necessary for the Working of Arithmetical Trigonometry.

By **Roger Rea**, N. P. Phi.

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Advertiſement.

AT the Hand and Pen near the Pump in *Crutched Fryers, London*, is Taught Wri-ting Arithmeticks (Vulgar and Decimal) and Merchants Accompts; Alſo Geometry, Trigonometry, Navigation, Aſtronomy, Dialling, Survey-ing, Gauging; with the Mea-ſuring of all ſorts of Artificers Work, the uſe of the Globes, and other Mathematical In-ſtruments,

By { ROGER And JOSEPH } REA, N. P.

To the READER.

THere having been ſome Enquiry for the uſe of the Sector to be made plain, I have endeavoured to do the ſame with all the exactneſs that I can, and alſo have compared it with the Plain Scale in ſeveral Geometrical Problems, and in all the Caſes of right lin'd Trigonometry, in the following Methods.

1. *There is given a true Deſcription of all the Lines both upon the Sector and the Plain Scale.*

2. *There is given ſeveral uſeful Problems in Practical Geometry, being very proper for the enſuing VVork.*

3. *There is a true Deſcription of the Line of Chords, Sines, Tangents and Secants, and alſo how to draw the ſame to any quantity of Degrees and Minutes.*

4. *There is given ſeveral Rules, whereby you may open the Sector to any Angle, or to any other Proportion required, being propoſed according to the ſeveral Lines thereon.*

5. *There is ſeveral Geometrical Proportions, performed both by the Sector and Plain Scale, and proved by ſuch Arithmetical Proportions as they will bear.*

6. *There is all the Cases of Right lin'd Trigonometry, both right Angled and oblique Angled, demonstrated both by the Sector and Plain Scale, and proved by Mr.* Gunter's *Scale.*

7. *There is all the preceeding Cases of right lin'd Trigonometry prov'd Arithmetically, so that any Person that understands Arithmetick may readily Answer any Question in right lin'd Trigonometry, without the help of any sort of Tables.*

8. *And because the Reader should not be to seek for proper helps in the VVorking of Arithmetical Trigonometry, there is given so much of Decimal Arithmetick, and the Extraction of the Square Root, as is necessary for the same, all being done with as much Plainness and Exactness, as the Subject will allow.*

And as few Authors appear in Publick without some Faults, so if any should be so curious as to find some small Faults therein, I hope they will pass a moderate Censure upon the same, I having taken all the Care I could to avoid them, for it was intended only for Learners, and doubt not, but taking a due Consideration, they may have the Benefit thereof to their Satisfaction; which is the hearty desire of him, who wisheth the Readers Welfare and Improvement

Roger Rea.

The Sector *and* Plain Scale *Compared.*

BEfore you begin the Operation of any Problem, it is neceſſary to underſtand the ſeveral Lines both upon the Sector and the Plain Scale, and afterwards you may proceed to the uſe of them in the following Problems of Geometry and right lin'd Triangles.

The Deſcription of the ſeveral Lines upon the Sector.

1. THE Line of equal Parts, commonly called the Line of Lines; is divided into 100 equal Parts, beginning at the Center, and divided firſt into 10 equal Parts, and each of theſe Parts, are ſubdivided into 10 other equal Parts, and are placed upon both Legs of the Sector, and upon the ſame ſide, they are Numbred 1, 2, 3, 4, *&c.* Or 10, 20, 30, 40, *&c.* unto either 10 or 100, coming very near the end of the Sector, and marked L.

Note,

☞ *Note*, They may be counted for 10, 100 or 1000, *&c.* and then 2 will signifie 20,200, or 2000, *&c.*

2. The Line of Chords, whose Radius is very near the length of one of the Legs, is also placed upon both Legs, and upon the same side of the Sector, beginning at the Center, and Numbred with 10, 20, 30, *&c.* unto 60, and marked with C.

3. The Line of Sines, is of natural Sines, being Projected from a Circle of the same Radius with the Chords, and placed upon both Legs of the Sector, and upon the same side, being numbred from the Center with 10, 20, 30, *&c.* to 90, and marked with S.

4. The Line Tangents is of common Tangents, projected from a Circle of the same Radius with the Sines, and plac'd upon both Legs of the Sector, and numbred from the center with 10, 20, &c. to 45. and marked with *T*.

5. Besides this, there is another small Line of Tangents placed upon both Legs of the Sector, beginning at about two Inches from the center, Numbred with 45, 50, &c. to 75, marked with *Tang.* the use of this Line is to supply the defect of the first Line of Tangents.

6. The Line of Secants, is of common Secants, and Projected from the afore-mentioned Radius, being placed upon both Legs of the Sector, and the same side, and numbred with 10, 20, 30, &c. to 75. being marked with *Sec.*

7. The

7. The Line of Polygons is grounded upon a Suppoſition, that every Circle is divided into 360 Degrees; therefore if you ſhould divide 360 by either 3, 4, or 5, &c. the Quotient will give you the Number of Degrees and Minutes in each of thoſe Parts, and the Points on the Line of Polygons will give you the Diviſions according to that Radius; and it is placed upon both legs of the Sector, and numbred with 12, 11, 10, &c. to 4, beginning at about 3 Inches from the Center, being marked *Polyg.* its chiefeſt uſe is to divide a Circle into any number of equal Parts.

8. The Sector being opened to its full length, on the back part thereof is a Line of Numbers, and upon one ſide a line of artificial Sines, and on the other ſide a line of artificial Tangents, theſe are for the working all the Caſes.

The Deſcription of the Lines upon the Plain Scale.

UPon one ſide is a line of equal Parts, next two lines of Chords of two ſeveral Radius's, the next a line of Sines, with a line of Secants over it, then a Line of Tangents, and a Line of Semi-Tangents, they are all Numbred with 10, 20, 30, &c. having each a Braſs Center fixed at the Units Place, and their proper Names marked to each Line.

2. Upon the other ſide of the Scale is two Diagonal Lines (or Lines of equal Parts) the one of them being twice ſo big as the other, and at each end there is one of the Diviſions

di-

divided into a 100 equal Parts, by Diagonal Lines being drawn through each tenth part.

Next I ſhall proceed to ſome neceſſary Rules in Practical Geometry, ſo far as may be uſeful in the following Problems.

DEFINITIONS.

1. A Point, or Punct, is that which cannot be divided into Parts, and is the end of a Mathematical Line, as the Point A. Plate 1. Fig. 1.

2. A Mathematical Line hath neither breadth or thickneſs, only length, and is made by the moving of a Point, and being conſidered in its ſelf, is either Regular or Irregular.

3. Regular, is either a right Line, or an Arch.

4. A right Line is the ſhorteſt diſtance between two Points, as the Line B. C. Plate 1. Fig. 1.

5. An Arch is not the ſhorteſt diſtance between two Points, but bendeth evenly, as D. E.

6. Irregular, is any crooked Line that bendeth unevenly, as F. G. Plate 1. Fig. 1.

7. Lines compared, are either Parellel or Inclining.

Parallels are of equal diſtance, and if infinitely produced, would never meet in the ſame Superfices, as A. B. and C. D. Plate 1. Fig. 2.

Inclining Lines are not of equal diſtance, and if produced would meet and form an Angle, as E F and G H, Plate 1. Fig. 2.

8. A right Lin'd Angle, is either right Angled or oblique Angled. A right Angle is formed when one right Line ſtandeth directly upon another

nother right Line, making the Angles on both sides equal, as A B D and C B D, an oblique Angle is either acute or obtuſe; an acute Angle is leſs than a right Angle, as the Angle A B E, is leſs then A B D; an obtuſe Angle is greater than a right Angle, as the Angle C B E is greater than the Angle C B D, Plate 1, Fig. 3.

Geometrical Problems

1. HOW to draw a right Line parallel to another right Line thro' a Point over the given right Line.

The right Line given is A B, and it is required to draw another right Line parallel to the ſame thro' the Point C. Plate 1. Fig. 4.

Set one foot of the Compaſſes in C. and with the other foot deſcribe an obſcure Arch juſt to touch the given Line A B, the Compaſſes at the ſame diſtance ſet one foot upon ſome part of the given Line as D, and with the other foot deſcribe another obſcure Arch on the ſame ſide the given Line that the Point is on, lay a Scale from the Point C to E, the upper Part of the Arch, and draw the right Line C E, which will be parallel to the given Line A B, as was required.

2. How to divide a given right Line into two equal Parts. The given right Line is A B, Plate 1. Fig. 5.

Set one Foot of the Compaſſes in A, and open them to above half the length of the given Line and deſcribe an Arch, the Compaſſes at

the ſame diſtance ſet one foot in B, and deſcribe another Arch to interſect the firſt Arch in C and D; lay a Scale from C to D, and draw a right Line which will cut the given right Line A B in E, dividing it into two equal Parts, as was required.

3. How to erect a Perpendicular upon the extream of a given right Line.

The right Line given is A B, and it is required to erect a perpendicular from A, Plate 1. Fig. 6.

Set one foot of the Compaſſes in A, and open them to a moderate wideneſs, and deſcribe an Arch from C, on the given Line A B, the Compaſſes at the ſame diſtance ſet one foot in O and make a Mark at E upon the Arch, and turning them over make another Mark upon the Arch as O, the Compaſſes ſtill at the ſame diſtance, ſetting one foot in E and O ſeverally, and deſcribe two Arches to interſect each other in D, lay a Scale from A to D, and draw the right Line A D, which will be Perpendicular to A B, as was required.

4. How to erect a perpendicular upon any part of a given right Line.

The right Line given is A B, and it is required to erect a Perpendicular from the Point C. Plate 1. Fig. 7.

Set one Foot of the Compaſſes in C, aud open them to a moderate wideneſs, and deſcribe an Arch to cut the given Line A B in D, the Compaſſes at the ſame diſtance, ſet one foot in D and deſcribe another Arch to cut the

firſt

first Arch in E, the Compasses still at the same distance, set one foot in E, the Point of intersection, and describe another Arch from C and over E the Point of Intersection, then lay a Scale from D to E, and draw an obscure right Line, to cut the last Arch in F; lastly, lay a Scale from C to F, and draw the right Line C F, which will be perpendicular to the given Line A B from the Point C, as was required.

5. How to let fall a perpendicular upon a given right Line, from a Point over it.

The right Line given is A B, and it is required to let fall a perpendicular upon the same, from the Point C being over it. Plate 1. Fig. 8.

Set one foot of the Compasses in C, and open them that they may cut the given Line A B in two places, as in D and E, there divide the distance D E into two equal Parts as in F, lay a Scale from F to C, and draw the right Line C F, which will be perpendicular to the given Line A B, as was required.

The Description of the Lines of Chords, Sines, Tangents and Secants.

IN the measuring the Parts of a Circle (which is very often required) there is no other Method, but by reducing the Circles to right Lines, therefore the Ancients did apply several strait Lines to a Circle, which came in competition with Arched Lines, and that several ways, *viz.* as they are drawn within a

Circle

Circle, through a Circle, or without a Circle.

1. If you suppose the Circumference of a Circle to be 360 equal Parts, they are called Degrees, and each Degree being divided into 60 other equal Parts, they are called Minutes; &c.

2. The Arch of a Great Circle is any part of the Circumference, and is counted in Degrees and Minutes, and is either greater or lesser in Proportion, to the Radius thereof.

3. The Radius of a Circle is but half the Diameter, or a right Circle drawn from the Center to the Circumference.

4. A Chord is a right Line drawn from one extream of an Arch to the other extream.

5. A Sine, is half the Chord of the double Arch.

6. The versed Sine, lieth between the Sine and the Circumference.

7. The Tangent toucheth the extream of the Diameter, and is Perpendicular unto it.

8. A Secant cutteth the Circumference of a Circle, being drawn from the center until it meet the Tangent.

How to draw the Lines of Chords, Sines, Tangents and Secants, and by the same reason to draw them to any other quantity of Degrees and Minutes, Plate 1. Fig. 9.

1. Take 60 d. from the Chords, and setting one foot upon A as a center, and describe the Circumference B E C and D.

2. Cross

2. Croſs the Circle at right Angles through the center A, by drawing the the two Diameters B A C and D A I.

3. Then divide I C into two equal Parts, as in E, which will be 45 Deg. diſtant from either I or C.

4. Then make C G equal to C E, and draw the right line E F G, which will cut the Diameter B A C in F:

5. At C the extream of the Diameter B A C, erect a perpendicular.

6 Lay a Scale from A the center to E, and draw a right Line to cut the perpendicular in H, then E C will be the Chord, E F the Sine, F C the verſed Sine, C H the Tangent and A H the Secant of 45 Degrees.

This Problem being the Foundation of the Lines of Rhumbs, Chords, Sines, Tangents and Secants; but its chiefeſt uſe is in a right lin'd, right angled Triangle, where any ſide may be made Radius.

The uſe of the Line of Sines.

1 HOW to find the Sine of any Arch.

The Radius of any Circle being equal to the Sine of *90* Deg. there is nothing more, but the taking of the quantity of Deg. and Minutes from the Sines on the Sector; but if it be greater or leſs than *90* Deg. let it be made a Radius between *90* and *90*, on the Sines between both Legs of the Sector, and that parallel diſtance being meaſured from the

the center upon the Sines, will be the Sine required

2: To open the Sector to Radius, or Sine of 90 Deg.

Take 90 Deg. from the center on the Sines, and place that diſtance upon 90 and 90, on the Sines between both Legs, and the Sector will be opened to 90 Deg. on the Sires to 60 deg. on the Chords, or to 45 deg. on the Tangets.

3. The right Sine of any Arch being given, to find the Rhadius, ſuppoſe the Sine of 46 deg. 30 m. given to find the Radius; from the center of the Sines take the given Sine between your Compaſſes, and open the Sector to the parallel of the given Sine of 46 deg. 30 m. between both Legs, the Sector being at the ſame Angle, take the diſtance between 90 and 90, on the Sines, and meaſured from the center of the Sines, will be Radius, or the Sine of 90 degrees,

4. The length of any right Sine being given, to find the quantity thereof in Degrees and Minutes.

Firſt, Open the Sector to Radius, or the Sine of 90 deg: and take the length of the given right Sine between your Compaſſes, and move them parallel upon both Legs of the Sector until both Feet reſt in like Sines, which being counted from the center, will be the quantity of Degrees and Minutes required, according to the length of the given right Sine:

5 The

5. The Sine of any Arch being given, to find the Chord of that Arch.

The Sine of any Arch being doubled, will give the Chord of that Arch, as suppose the Sine of 30 deg: from the Center of the Sines, take 15 deg. between your Compasses, it being half the Sine of 30 deg: and set one Foot upon the Center of the Chords, they being turned twice over, will reach the Chord of 30 deg. answering to the given right Sine.

6. The right Sine of an Arch being given, to find the versed Sine thereof.

Suppose the Sine of 60 deg: given, to find the versed Sine thereof:

First open the Sector to Radius, or the Sine of 90 deg. then take the Complement of the given Sine 60 deg. to 90 deg. and the remains will be 30 deg. the Sector continued at the same Angle, the parallel between 30 deg. and 30 deg. taken between your Compasses, and measured from 90 deg. on the Sines towards the Center, the other Foot will reach to 30 deg. from the Center thereof, which will be 60 deg. counted from the Sine of 90 deg. towards the Center.

But you may suppose an Arch whose versed Sine is 140 deg. which being above 90 deg. the excess above 90 deg. being 50 deg. therefore the versed Sine is equal to 90 deg. and 50 deg. put together.

7. The Chord of an Arch being given, to find the right Sine of 90 deg. Suppose the Chord of 80 deg. given, to find the Chord of 80 deg. or the Radius or Sine of 90 deg. From

From the Center of the Sines, take 40 deg. the half of the given Chord 80 deg. between your Compaſſes and make it a parallel Sine between 40 deg. and 40 deg. the Sector being continued at that Angle, the diſtance between 90 deg. and 90 deg. taken and meaſured from the Center will be the Radius of 90 deg. and the diſtance between 80 deg. and 80 deg. being meaſured as before will be 80 deg.

8. How to open the Sector to the quantity of any given Angle.

From the Center of the Sines, take half the quantity of the given Angle between your Compaſſes, and make it a parallel between 30 deg. and 30 deg. or 50 and 50 on the Line of Lines (which is the ſame) either of them will open the Sector to the quantity of the Angle required.

9. How to open the Sector to a right Angle, by the Line of right Sines.

From the Center of the Line of Sines take 90 deg. between your Compaſſes, and make it a parallel Sine of 45 deg. and 45 deg. or if from the Center of the Sines, you take 45 deg. and make it a parallel between 30 deg. and 30 deg. on the Sines, either of them will open the Sector to a right Angle.

10. How to open the Sector to a right Angle by the Line of Lines.

From the Center of the Line of Lines, take the whole Radius to 10, between your Compaſſes, and place that diſtance from 8 on the Line of Lines upon one Leg, and open the Sector untill the other Foot will fall upon 6 on the

the Line of Lines upon the other Leg, and the Sector will be opened to a right Angle as before.

I shall proceed now to the use of the Sector, in some Geometrical Problems, and prove each of them by the Plain Scale.

1. HOW to divide a given right Line into any Number of equal Parts.

Suppose you were to divide a right Line into as many equal Parts as one Leg of the Sector contains.

Take the length of one Leg of the Sector between your Compasses from the Center of the Line of Lines, and make it a parallel between 10 and 10, of the same Line of Lines, the Sector being continued at that Angle, the parallel distance between every like Number upon both *Legs*, will divide the given Line into the same Number of equal Parts, as the length of one Leg of the Sector from the Center contains.

EXAMPLE, Plate 1. Fig. 10.

Suppose you were to divide the right Line A B into 5 equal Parts.

1. By the Sector.

Take the length of the given right Line between your Compasses, and make it a parallel of 5 and 5, between both Legs of the Sector, upon the Line of Lines, and the parallel di-

ſtance between 5 and 5 will be 5 equal Parts, and 4 and 4 will be 4 equal Parts, &c. and the given Line A B ſhall be divided into 5 equal Parts as required.

Note, That the dividing a right Line into any Number of equal Parts, will form an Equilateral Triangle, as A B C.

(2.) To perform the ſame by the Plain Scale, Plate 1. Fig. 11.

From one of the Extreams of the given Line A B, as at A, draw the Line A C making any Angle, and at B the other Extream of the given Line, draw the Line B D parallel to A C, and upon the two Lines A C and B D place 4 equal Parts, (that is leſs by one than the given Line A B was to be divided into) and Number them from A towards C, and from B towards D, with 1, 2, 3 and 4, and draw obſcure right Lines from 1 to 4, and from 2 to 3, &c. which croſſing the given Line A B will divide it into 5 equal Parts, as was required.

2. How to increaſe or diminiſh a given right Line according to any Proportion required.

Example, Plate 1. Fig. 12.

Suppoſe the right Line A B to be 6 equal Parts, and it is required to increaſe it to 8 of thoſe Parts.

(1.) By the Sector.

From the Center of the Line of Lines take 6 Parts the length of the given Line A B, between your Compaſſes, and make it a parallel between 6 and 6 upon the Line of Lines, then

if you take the parallel distance between 8 and 8, and measure it from the Center of the Line of Lines, and you will find it to be 8, the length of the required Line.

But if it was required to diminish it to 4 of those Parts, you must make it a parallel between 6 and 6 as before, and the parallel distance between 4 and 4, taken and measured from the Center of the Line of Lines, will be 4, the length of the required Line diminishing.

(2.) By the Plain Scale. Plate 2. Fig. 13.

Draw a right Line of any moderate length, and from a Scale of equal Parts take 6, and place it from A to B, and from the same Scale take 2, and place it from B to C, it will make 8, the length of the required Line.

But if you were to diminish the given Line A B to 4, it is but taking 2 from the same Scale of equal Parts, and placing it from B to D and it is done.

3. Two Lines or Numbers given, to find a third Line or Number in continual Proportion, either increasing or diminishing.

Example, Plate 2. Fig, 14.

Suppose two Lines to be given, as A 18, and B 24, to find a third proportional Number or Line increasing.

(1.) By the Sector.

From the Line of Lines take 24 from the Center, and make it a parallel between 18 and 18 on the Line of Lines, (between both Legs) the Sector being continued at that Angle,

 take

take the nistance of 24 and 24, between your Compasses, and measure it from the Center o the Line of Lines, you will find it to be about 32, the third proportional Number increasing.

But if it were diminishing,

Then you must make the shortest given Line or Number a parallel upon the Line of Lines between longest of the given Lines or Numbers, then take the parallel distance between the shortest of them between your Compasses,and measure it from the Center of the Line of Lines, will be the third proportional Line or Number diminishing.

(2.) By the Plain Scale, Plate 2. Fig. 15.

Draw two right Lines making any Angle, as G A E, then from a Line of equal Parts take the length of A 18 between your Compasses,and place it from A to C, also take 24 and place it from A to B, and from A to D, then draw the obscure right Line C B, and from the Point D draw the obscure Line D E parallel to C B, so A F will be the third proportional Number increasing. As A C 18. A B 24 : : A D 24. A F 32.

But if it were diminishing, let the two Numbers be A F 32, and A D 24.

From the same equal Parts take A F 32 and place it from A to F, and A D 24, and place it from A to D, and from A to B, then draw the obscure right Line F D, and from the Point D draw the obscure Line B C parallel to F D, and the Line A C will be the third proportional Line or Number diminishing.

As

As A F 32. A D 24: A B 24.: A C 18.

Note, That you form an Angle by drawing two right Lines, as for increasing.

4. Three Lines or Numbers given, to find a fourth proportional Line or Number, either increasing or diminishing.

Example, Plate 2. Fig. 16.

Let the three Lines given, be A 24, B 28, and C 36.

(1.) By the Sector increasing.

From the Center of the Line of Lines, Note A 24 upon one Leg, and B 28 upon the other Leg, and from the Center of the Line of Lines take B 28 between your Compasses and make it a parallel between 24 and 24 upon both Line of Lines, the Sector being continued at that Angle, take the distance between 36 and 36, the given *Line*, between your Compasses, and measure it from the Center of the *Line* of *Lines*, and it will be 42, the fourth proportional *Line* required, increasing.

But if you would have it diminishing, make A 42, B 36, and C 28.

Then from the Center of the *Line* of Lines, Note A 42 upon one *Leg*, and B 36 upon the other *Leg* of the Sector; then from the Center of the *Line* of *Lines* take the length of the second *Line* B 36 between your Compasses, and make it a parallel between 42 and 42 upon the *Line* of *Lines*, the Sector being continued at that Angle, take the parallel distance between 28 and 28 of the Line of *Lines* between your Compasses and measure it from rhe Center of the

the Line of Lines, you will find it to be 24, the fourth proportional Line diminishing.

(2:) By the Plain Scale increasing, Plate 2. Fig. 17.

Draw two right Lines making any Angle, as G F E, and from a Scale of equal Parts take A 24 and place it from F to D, and also B 28 and place it from F to H, and C 36 and place it from F to I, then draw the obscure Line D H, and from the Point I, draw the obscure Line I K parallel to D H, so the distance F K taken between your Compasses and measured upon the same equal Parts will be 42, the fourth proportional Line required, increasing.

For, as F D 24, F H 28, F I 36. F K 42.

But if you would have it diminishing, you must make A 42, B 36, and C 28.

Draw two right Lines making any Angle, as G F E, and from a Scale of equal Parts take A 42 and place it from F to K, and B 36 and place it from F to I, and also C 28 and place it from F to H, then draw the obscure Line K I, and from the Point H draw the obscure Line H D parallel to K I, then F D taken between the Compasses and measured upon the same equal Parts will be 24, the fourth proportional Line required, diminishing.

For, as F K 42. F I 36: F H 28, F D 24.

5. How to divide a given right Line into two Parts, in such Proportion as one given right Line hath unto another.

Example, Plate 2. Fig. 18.

Let

Let the right Line given to be divided be 40, to be divided in ſuch Proportion as A 20 hath to B 30.

(1.) By the Sector.

From the Center of the Line of Lines, Note 50 the length of the two propoſed Lines in one Sum, and make 40 the Line to be divided a parallel between 50 and 50, upon both the Line of Lines, and the parallel diſtance between 30 and 30, on the Line of Lines taken between your Compaſſes and meaſured from the Center of the Line of Lines will be 24, one part of the given right Line, and the parallel diſtance between 20 and 20 of the Line of Lines, being taken and meaſured as before, will be 16, the other Part thereof.

(2.) By the Plain Scale, Plate 2. Fig. 19.

Take A *B* 40 from a Scale of equal Parts, and draw a right Line according to that length, and from A draw another right Line making any Angle as *B* A E, then take the length of the Line *B* 30 from the ſame equal Parts and place it from A to D, and from the ſame equal Parts take A 20, and place it from D to E, and draw the obſcure Line B E, then from the Point D draw another obſcure Line, parallel to the firſt obſcure Line *B* E, as D C, which Point C will divide the given right Line A B into two Parts, in ſuch Proportion as the Line A 20 hath to *B* 30, as was required.

For, as A E 50. A *B* 40. A D 30. A C 24.

Again, as A E 30. A *B* 40. D E 20. C *B* 16.

I shall now proceed to all the Cases of right Lin'd Trigonometry, VVorking the several Opperations both by the Sector and Plain Scale, and prove them by either of Mr. Gunter's *Scales.*

1. TRigonometry sheweth how to find out the several Parts in a right Lin'd Triangle, as the length of the sides and the quantity of the Angles.

2: A Triangle consisteth of three Sides and three Angles.

3. An Angle is formed by the Intersection of two right Lines, and is either a right Angle or an oblique Angle.

4. A right Angle containeth just 90 Degrees, but of oblique Angles there are two sorts, *viz.* Obtuse and Accute, an obtuse Angle containeth more than 90 deg. but an accute Angle is less than 90 degrees.

5. In all right Lin'd Triangles the three Angles being added together, will be equal to 180 Degrees.

6. The Complement of any Angle to a right Angle, is what that Angle wants of 90 Degrees, but the Complement of any Angle to a Semi-Circle, is what it wants of 180 degrees.

7. In a right Angled Triangle, the side opposite to the right Angle is called the Hipothenuse,

nuſe, and the two Legs the Baſe, or perpendicular at Pleaſure.

8. In all right Angled Triangles, two things being given beſides the right Angle, provided one of them be a Side, are ſufficient to find the reſt; but in an oblique Angled Triangle, there muſt be three Things given, and one of them a Side, or the three Sides, but not the three Angles, for the Angles of a right Lin'd Triangle being only given, the Sides cannot be found.

In the Doctrine of right Angled right Lin'd Triangles, they are divided into Seven Caſes, but I make uſe of but Four Caſes, becauſe they properly admit of no more.

1. *Note*, That the Sides and Angles of a Triangle, are diſtinguiſhed by the Letters of the Alphabet, for two Letters denote a Side, and three Letters an Angle, and the middle Letter always gives the Angular Point.

2 *Note*, That the Sides of all right Lin'd Triangles, are meaſured by a Scale of equal Parts, and the Angles are meaſured by a Line of Chords.

C A S E the Firſt.

The Baſe and Angles given, to find the length of the Perpendicular and the Hipothenuſe.

D *Ex-*

Example, Plate 2. Fig. 20.

In the right Angled right Lin'd Triangle A B C, right Angled at B, there is given the Base A B 85 Lea. the Angle at the Base B A C 35 deg: 36, the Angle at the Perpendicular A C B 54 deg. 24, to find the length of the perpendicular B C, and the Hipothenuse A C.

(1:) By the Sector.

From the Center of the Line of Lines, take the length of the Base A B 85 Lea. between your Compasses, and make it a parallel at the Sine of its opposite Angle A C B 54 deg. 24, between both Legs of the Sector, the Sector being continued at the same Angle, take the distance of 90 and 90 of the Sines, and measuring it from the Center of the Line of Lines, will be the length of the Hipothenuse A C 105 Lea. the Sector continued still at the same Angle, take the distance of the Sine of the Angle B A C 35 deg. 36, between your Compasses, and measure that distance from the Center of the Line of Lines, will be the length of the perpendicular B C 61, Lea.

2. By the Plain Scale.

From a Scale of equal Parts draw the length of the Base A B 85 Lea. at B erect a perpendicular, then take Radius (or 60) from the Chords, and setting one Foot in A describe an Arch from D on the Line A B, from the same Chords take the quantity of the Angle B A C 35 deg. 36, between your Compasses, and place it from D to E, lay a Scale from A to E and draw a right Line to cut the perpendicular

in

in C, and the diſtances A C and B C being taken ſeverally between your Compaſſes, and meaſured upon the ſame equal Parts, will be the length of the Hipothenuſe A C 105 Lea. and the perpendicular B C 61 Lea. agreeing exactly with the Sector.

By the Gunter's Scale.

1. To find the length of the perpendicular B C, as S. Angle A C B 54 deg. 24.. Logm. A B 85 Lea.: : S. Angle B A C 35 deg. 36 Logm. B C 61 Lea.

2: To find the length of the Hipothenuſe A C, as S. Angle A C B 54 deg. 24, Logm. A B 85 Lea : : R.. Logm. A C 105. Lea.

Caſe the Second.

The Hipothenuſe and Angles given to find the length of the Baſe and the Perpendicular.

Example, Plate 2. Fig. 21.

In the right Angled right Lin'd Triangle A B C, right Angled at B, there is given the Hipothenuſe A C 99 Lea. the Angles A C B 58 deg. 30. and B A C 31 deg. 30. to find the length of Baſe A B, and the Perpendicular B C.

1. By the Sector.

From the Center of the Line of Lines take length of the Hipothenuſe A C 99 Lea. between your Compaſſes, and make it a parallel between 90 and 90 on the Line of Sines, the Sector being continued at that Angle, take the parallel Sine of the Angle A C B 58 deg. 30,

 be-

between your Compasses, and measure that distance from the Center of the Line of Lines, will be above 84 Lea. the length of the Base A B, and the parallel distance of the Angle B A C 31 deg. 30, being taken and measured as before, will be the length of the perpendicular B C 51 Lea. as was required.

2. By the Plain Scale.

Draw a right Line from A at any moderate length, and from the Chords take Radius (or 60 deg.) between your Compasses, set one Foot in A and describe an Arch from D, from the same Chords take the quantity of the Angle B A C 31 deg. 30, between your Compasses, and place it upon the Arch from D to E, lay a Scale from A to E and draw the right Line A C 99 Lea. then from C let fall a perpendicular, to cut the Line drawn from A in B. so A B will be about 84 Lea. the length of the Base; and B C 51 Lea. they being both measured upon the same Scale of equal Parts and it is done.

By the Gunter's Scale.

1. To find the length of the Base A B.

As R.. Logm. A C 99 Lea :: S: Angle A C B 58 deg. 30.. Logm. A B 84 Lea.

2. To find the length of the Perpendicular B C.

As R.. Logm. A C 99 Lea :: S. Angle B A C 31 deg. 30.. Logm: B C 51 Lea.

Cas

Case the Third.

The Base and Perpendicular given, to find the quantity of the Angles and the length of the Hipothenuse:

Example, Plate 2. Fig, 22.

In the right Angled right Lin'd Triangle, A B C right Angled at B, there is given the Base A B 71 Lea; and the Perpendicular B C 63 Lea. to find the quantity of the Angles B A C and A C B, and the length of the Hipothenuse A C.

1. By the Sector, This requires a double Operation.

(1.) To find the quantity of the Angles B A C and A C B.

From the Center of the Line of Lines, note, the length of the Base A B 71 Lea. and from the Center of the same Line of Lines take the length of the Perpendicular B C 63 Lea. between your Compasses, and make it a parallel between 71 and 71 on the Line of Lines, the Sector being continued at the same Angle; take the parallel Radius of the Line of Lines between your Compasses, and being measured from the Center of the Line of Tangents, will be about 41 deg. 37, the quantity of the Angle B A C, which being substracted from 90 deg. will leave 48 deg. 23, the quantity of the Angle A C B.

(2. To find the length of the Hipothenuse A C.

By

By the Line of Lines open the Sector to a right Angle, then from the Center of the Line of Lines, note, the length of the Base A B 71 Lea. upon one of the Legs, and the length of the perpendicular B C 63 Lea. upon the other Leg, the Sector being continued at a right Angle, take the distance between both Legs of the Sector from 71 to 63 between your Compasses, and being measured upon the Line of Lines from the Center, will be 94 Lea. the length of the Hipothenuse A C, as was required.

2. By the Plain Scale.

From a Scale of equal Parts draw the length of the Base A B 71 Lea. and at B erect the perpendicular B C 63 Lea. from the same equal Parts; lay a Scale from A to C, and draw a right Line as A C, which being measured upon the same equal Parts will be 94 Lea. the length of the Hipothenuse A C; the two Angles are measured by the Line of Chords, as before.

By the Gunter's Scale.

1. To find the quantity of the Angle A C B.

As Logm. A B 71 .. Lea. R :: Logm. B C 63 Lea. Tang. Angle B A C 41 deg. 37, which being substracted from 90 deg. leaves the quantity of the Angle A C B 48 deg. 23.

2. To find the length of the Hipothenuse A C.

As S. Angle B A C 41 deg. 37 .. Logm. B C 63. Lea :: R.. Logm. A C 94 Lea:

Case

Case the Fourth.

The Hipothenuse and one of the Legs given (either the Base or Perpendicular) to find the Angles and the other Leg.

Example, Plate 2: Fig: 23:

In the right Angled right Lin'd Triangle, A B C right Angled at B, there is given the Hipothenuse A C 83 Lea. and the Base A B 74 Lea. to find the quantity of the Angles B A C and A C B, and the length of the Perpendicular B C.

1. By the Sector, This likewise requires a double Operation.

(1.) To find the quantity of the Angles B A C and A C B.

From the Center of the Line of Lines take the length of the Hipothenuse A C 83 Lea. between your Compasses, and make it a Parallel between the Radius of the Line of Lines, then from the Center of the Line of Lines take the length of the *Base* A *B* 74 Lea. between your Compasses, and with that distance move them along upon the Line of Sines, upon both Legs of the Sector until the Feet rest in like Sines, which will be at 63 deg: 4, the quantity of the Angle A C *B*, (being opposite to the Base A *B*) which being Substracted from 90, leaves 26 deg. 56, the quantity of the Angle *B* A C.

(2.) To find the length of the Perpendicular *B* C.

Open

Open the Sector to a right Angle by the Line of Lines, and from the Center thereof, upon one of the Legs, note the length of the *Base* A B 74 Lea. the Sector being still continued at a right Angle, from the Center of the Line of Lines, take the length of the Hipothenuse A C 83 Lea. between your Compasses, and set one Foot upon the length of the Base A B 74 Lea. that was noted, and the other Foot being turned towards the Center of the Line of Lines, it will rest upon 37 Lea. the length of the Perpendicular B C, as was required.

2. By the Plain Scale.

From a Scale of equal Parts draw the length of the Base A B 74 Lea: and at B erect a perpendicular, from the same Scale of equal Parts take the length of the Hipothenuse A C 83 Lea. between your Compasses, and setting one Foot in A, with the other Foot make a Mark upon the Perpendicular as at C, lay a Scale from A to C and draw a right Line to cut the Perpendicular in C, so that B C being measured upon the same equal Parts will be 37 Lea. the length of the Perpendicular as required, the Angles are measured as before.

By the Gunter's Scale.

1, To find the quantity of the Angles A C B and B A C.

As Logm: A C 83, Lea.. R:: Logm. A B 74 Lea.. S. Angle A C B 63 deg 4, which being Substracted from 90 deg. leaves 26 deg. 56, the quantity of the Angle B A C.

2 T

2. To find the length of the Perpendicular B C.

As R.. Logm. A C 83 Lea:: S. Angle B A C 26 deg. 56.. Logm. B C 37 Lea.

Of Oblique Right Lin'd Triangles.

IN the Doctrine of oblique right Lin'd Triangles, there are Five Cases, but according to what is given and required, they admit but Four.

Case the First or Fifth.

Two Sides with an Angle opposite (or adjacent) to one of them given, to find the other two Angles and the third Side.

Example, Plate 2. Fig. 24.

In the oblique right Lin'd Triangle A B C, there is given the side A C 56 Lea. the side B C 42 Lea. and the Angle B A C 29 deg. 30. (opposite to B C) to find the quantity of the other two Angles A B C and A C B, and the length of the Side A B.

Note, That when any Angle is obtuse (or above 90 deg.) you must always work by its Complement to 180 deg.

1. By the Sector, This Case requires a double Operation.

(1) To find the third Side A B.

Open the Sector to the quantity of the given Angle B A C 29 deg. 30 by the Line of Sines, and

and from the Center of the Line of Lines, note the length of the adjacent Side A C 56 Lea upon one of the Legs, keep the Sector at the same Angle, and from the Center of the Line of Lines take the quantity of the other Side B C 42 Lea. between your Compasses, and set oneFoot in the Number that was noted upon one of the Legs, and let the other Foot fall on the Line of Lines upon the other Leg, and it will reach to about 80 Lea. But you are to take Notice, that that Foot will cross the other Leg in two places, and which of the two places will be the length of the required side must be determined by the quantity of the Angle opposite to that side, for if the opposite Angle be acute, that part of the Line of Lines towards the Center, where the Compasses cross the other Leg will be the length of the required Side; but if the Angle be obtuse then that Part opposite to the Center where the other Foot of the Compasses cross the other Leg, will be the length of the required side, therefore this is called by most, *The Doubtful Case*.

(2) To find the quantity of the Angle A B C:

From the Center of the Line of Lines, take the length of the Side B C 42 Lea. between your Compasses, and make it a parallel distance to the Sine of its opposite Angle B A C 29 d 30, then from the Center of the Line of Lines take the length of the Side B C 56 Lea. (adjacent to the given Angle) between your Compasses

passes, (the Sector being continued at the same Angle) and moving them parallel along the Sines on both Legs until they rest in like Sines, which will be about 41 deg. 2, the quantity of the Angle A B C.

To find the quantity of the Angle A C B, you may add the two Angles B A C 29 deg. 30, and A B C 41 2 together, they will make 70 deg. 32, which being Substracted from 180 deg. will leave 109 deg. 28, the quantity of the Angle A C B.

2. By the Plain Scale.

Draw a right Line from A at any moderate length, and from the Chords take the Radius (or 60 deg.) and setting one Foot in A, and from D on the right Line describe an Arch, from the Chords take the quantity of the Angle B A C 29 deg. 30 between your Compasses and place it upon that Arch from D to E, lay a Scale from A to E and draw the right Line A C 56 Lea. and from the same equal Parts take the length of the Side B C 42 Lea. between your Compasses, and make a Mark upon the Line drawn from A as at B, (the Angle A C B being obtuse) you must place the Foot of the Compasses at farthest distance from A, and it is done.

Note, That when one Foot of the Compasses is in C, the other Foot will cut the Line drawn from A in two places, but its opposite Angle being obtuse, you must place the Letter B at the farthest distance from A.

By the Gunter's Scale.

1. To find the quantity of the Angle A B C. As Logm. B C 42 Lea.. S. Angle B A C 29 deg. 30.:: Logm. A C 56 Lea.. S. Angle A B C 41 deg. 2.

2. To find the length of the third Side A B. As S. Angle B A C 29 deg. 30.. Logm. B C 42 Lea:: Sc. Angle A C B 109 deg. 28, to 180 deg. or 70 deg. 32.. to Logm. A B 80 Lea.

Case the Second, or Sixth.

Two Angles of an oblique right Lin'd Triangle with one of the Sides given, to find the third Angle, and the other two Sides.

Example, Plate 3. Fig, 25.

In the oblique right Lin'd Triangle A B C, there is given the Angle A C B 115 deg: 24 the Angle B A C 28 deg. 30, with the length of the Side A C 75 Lea. to find the quantity of the Angle A B C, and the length of the two Sides A B and B C.

1 *Note*, That the quantity of the Angle A B C is found by adding the two given Angles together, and Substracting their Sum from 180 deg. the Remains will be the quantity of the required Angle A B C.

2 *Note*, That when an obtuse Angle is given, you must always Work by its Complement to 180 deg.

1. By the Sector.

From the Center of the Line of Lines take the length of the given Side A C 75 Lea. between

ween your Compaſſes and make it a Parallel etween the Sines of its oppoſite Angle A B C 6 deg. 6, (found by the Subſtraction) the ector being continued at that Angle, take the iſtance between the Sines of the Angle B A C 8 deg. 30 between your Compaſſes, and meaure that diſtance from the Center of the Line f Lines, and it will be 61 Lea. the length of its ppoſite ſide B C, and the diſtance taken between 4 deg. 36 (the Complement of the Angle ACB 15 deg. 24, to 180 deg.) it will be 115 Lea. e length of the Side A B, as was required.

2. By the Plain Scale.

From a Scale of equal Parts draw the length the given Side A C 75 Lea. then take Radius r 60 deg.) from the Line of Chords, and tting one Foot of the Compaſſes in C, and ith the other deſcribe an Arch from D, the ngle A C B 115 deg. 24 being obtuſe, take 7 deg: 42, one half thereof between your ompaſſes, and ſet one Foot in D, turn them vice over to E, lay a Scale from C to E and aw a right Line at any moderate length, then om the ſame Chords take the Radius beween your Compaſſes, and ſetting one Foot A, and deſcribe another Arch from F, and om the ſame Chords take the quantity of e Angle B A C 28 deg. 30 between your ompaſſes, and place it from F to G on the ſt Arch, lay a Scale from A to G and draw ight Line that will cut the Line C E in B, d it is done.

That

☞ That the Lines A B and C B are measured upon the same equal Parts, as the Line A C.

By the Gunter's Scale.

1. To find the length of the Side A B.

As S. Angle A B C 36 deg. 6.. Logm. A C 75 Lea.: Sc. Angle A C B 115 deg. 24, to 180 deg. or 64 deg. 36.. Logm. A B 115 Lea.

2. To find the length of the Side B C.

As S. Angle A B C 36 deg. 6 .. Logm. A C 75 Lea :: S. Angle B A C 28 deg. 30 .. Logm. B C 61 Lea.

Case the Third. or Seventh.

Two Sides with their contained Angle given to find the other two Angles and the third Side.

Example, Plate 3. Fig. 26.

In the oblique right Lin'd Triangle A B C there is given, the Side A B 357 Lea. the Side A C 273 Lea. with their contain'd Angle B A C 33 deg. 36, to find the other two Angles A B C and A C B, and the length of the third Side B C.

1. By the Sector.

This Case requires a double Operation.

(1) To find B C the length of the Third Side.

Open the Sector to the quantity of the given Angle B A C 33 deg. 36, by either the Line of Lines, or the Line of Sines, then from the Center of the Line of Lines, note the two given Sid

Sides A B 057 Lea. and A C 357 Lea. upon both Legs of the Sector, the Sector being continued at the ſame Angle, take the diſtance between the two Sides that was noted between your Compaſſes, and meaſure it from the Center of the Line of Lines, and you will find it to be 204 Lea. the length of the third Side B C.

(2) To find the quantity of the Angle ABC oppoſite to the ſide A C.

From the Center of the Line of Lines take the length of the ſide *B* C 204 Lea. between your Compaſſes, and make it a parallel Sine of its oppoſite Angle B A C 33 deg: 36 between both Legs of the Sector, the Sector being continued at that Angle, from the Center of the Line of Lines take the length of the ſide A C 273 Lea. between your Compaſſes, and with that diſtance move them parallel upon the Sines on both Legs until they reſt in like Sines, which will be about 49 deg. 22, the quantity of the Angle A B C.

The quantity of the Angle A C B is found by adding the given Angle B A C 33 deg. 36, and the Angle A B C 49 deg. 22, making together 82 deg. 58, which being Subſtracted from 180 deg. leaves 97 deg. 2, the quantity of the Angle A C B.

By the Plain Scale.

From a Scale of equal Parts draw the length of the Side A B 357 Lea. and from the Chords take Radius (or 60 deg.) between your Compaſſes, and ſet one Foot in A and deſcribe an

Arch

Arch from D, from the same Chords take th quantity of the given Angle *B*AC 33 deg. 36, be tween your Compasses and place it from D t E, lay a Scale from A to E and draw a right Lin equal to the other given side A C 273 Lea. from the same equal Parts, lay a Scale from B to C and draw the right Line B C and it is done.

By the Gunter's Scale.

1. To find the quantity of the two Angle A B C and A C B.

A B 357 Lea.	B A C 180 deg
A C 273 Lea:	33. 36.
Their Sum —— 630 Lea.	**The Sum of ABC and ACB 146.24**
Their Difference —— 84 Lea:	**Their half Sum —— 73. 12**

Then, as Logm. of A B and A C 630 Lea. Logm. of their Dif. 84 Lea :: Tang: half A B C and A C B 73 deg. 12 .. Tang. 23 deg. 50 which being added to 73 deg. 12, makes th quantity of the Angle A C B 97 deg. 2; and being Substracted from 73 deg. 12 leaves 49 deg. 22, the quantity of the Angle A B C.

2. To find the length of the Side B C.

As S. Angle A B C 49 deg. 22 .. Logm. A C 273 Lea : : S. Angle *B* A C 33 deg 36 .. Logm B C 204 Lea,

Case the Fourth, or Eighth.

The Three Sides of an oblique right Lin'd Triangle given, to find either of the Angles:

Ex-

Example, Plate 3: Fig: 27.

In the oblique right Lin'd Triangle A B C, there is given the Side *A* B 584 Lea. the Side A C 398 Lea. and the Side B C 268 Lea. to find the quantity of the Angle A C B.

1. By the Sector.

From the Center of the Line of Lines, note the length of the two Sides A C 398 Lea. and B C 268 Lea. (which contain the required Angle A C B) upon both Legs of the Sector, and from the Center of the Line of Lines take the length of A *B* 584 Lea. the Side opposite to the required Angle A **C** *B* between your Compasses, and with that distance setting one Foot in 398 and open the Sector until the other Foot will rest upon 268, continue the Sector at that Angle, and take the distance between 30 and 30 of the Sines between your Compasses, and measure it from the Center of the Line of Sines will be about 60 deg. 39. half the quantity of the required Angle, which being doubled will be 121 deg. 18, the quantity of the required Angle, A C *B*.

2 *By* the plain Scale.

From a Scale of equal parts, draw the length of the Side *A B* 584 Lea. from the same equal Parts, take the length of the Side *A* C 398 Lea. and setting one Foot in *A* describe an *A*rch over the given Line A *B*, and from the same equal Parts take *B* C 268 *L*ea. and setting one Foot in B describe another Arch to Interfect the first Arch in C, lay a Scale from A to C, and from B

to C feverallay, and draw two right Lines to cut each other in G, and it is done.

In all right Lin'd Triangles the Proportion is.

As the true Base is to the Sum of the other two Sides, so is the difference of those two Sides, to the alternate Base, Substract the lesser of those Bases from the greater, and in the midst of the Remains will fall a Perpendicular, and reduce the oblique Triangle into two right Angled Triangles, in each of which there will be given the Hipothenuse and one Leg to find the Angles.

Note, That according to this Rule you may make any side the true Base, for if you make the longest side the true Base, the Perpendicular will fall within the Triangle, but if you make either of the shortest sides the true Base, the Perpendicular will fall without the Triangle.

By the Gunter's Scale.

1. To find the length of the Alternate Base, by making the longest side the true Base.

As Logm. A B 584 Lea .. Logm. A C and B C 666 Lea :: Logm. of their Difference 130 Lea.. A D 148 Lea. which being Substracted from he true A B 584 Lea. will leave 436 Lea. or B D, the half of it is B E 218 Lea. at E erect the Perpendicular E C, and reduce the oblique Triangle A B C, into two right Angled Triangles, in the right Angled Triangle A C E, you have the Hipothenuse A C 398 Lea. and the Leg A E 366 Lea. (by adding A D 148 Lea. to

to D E 218 Lea. together) to find the Angle A C E, and in the right Angled Triangle B C E you have given the Hipothenuſe B C 268 Lea. and the Leg B E 218 Lea. to find the Angle B C E.

2. To find the quantity of the Angle ACE. As Logm. A C 398 Lea.. R:: Logm. A E 366 Lea.. S. Angle A C E 66 deg. 52.

3. To find the quantity of the Angle BCE. As Logm. A C 268 Lea.. R::. Logm. B E 218 Lea . S. Angle B C E 54 deg. 25, unto which if you add the quantity of the Angle A C E 66 52. it will give the quantity of the required Angle A B C 121. 17.

There is another way to find the required Angle A C *B* at one Operation, as thus. Plate 3. Fig. 28.

You muſt add all the three Sides together and half that Sum, and from that half Sum Subſtract the three given Sides ſeverally, but firſt, the ſide oppoſite to the given Angle, then ſay,

As the Logarithm of half the Sum of the three ſides is to the Logarithm of the firſt Remainder, ſo is the Logarithm of the other two Remainders, ſeverally to the Square of a Tangent, whoſe half Sum (or Root) will give the Tangent of half the required Angle.

Note, That you muſt take the Complement Arithmetick for the firſt two Numbers.

To find the quantity of the Angle ACB.

AB 584 Lea.	The ½ Sum 625 Lea.	The ½ Sum 625 Lea.
AC 398 Lea.	AC —— 398——	BC—— 268.
BC 268. Lea.	2d Rem. 227.	Third Rem. 357.

Their Sum 1250 Lea.

The ½ Sum 625 Lea.
AB Substr. 584 Lea.

First Rem: 41 Lea.

As the Logm. of half the Sum of the three sides 625 Lea. Co. Ar.		7.204119.
Is to the Logm. of the first Remainder 41 Lea. Co. Ar.		8.387215.
So is the Logm. of the other two Remainders severally	227 Lea.	2.356026.
	357 Lea.	2.552668.
To the Square of the Tangent —— —— ——		20.500028.
The Root or half of which is the Tang. of 60 deg: 39.		10.250014.
Which being doubled is the quantity of the Angle ACB 121 18.		

Having any two Sides of a right Angled right Lin'd Triangle given, the third Side may be found by the Extraction of the square Root.

1. When you have both the Legs given, to find the length of the Hipothenuse, you must square the length of both the Legs severally, and set them together, and from that Sum extract the square Root, and that Root will be the length of the Hipothenuse required.

Example.

In a right Angled right Lin'd Triangle, there is given the length of one of the Legs 48 Lea. and the length of the other Leg 36 Lea. to find the length of the Hipothenuse.

48

48 Lea.	36 Lea.	
48.	36.	
384	216	3600 (60 Lea. Hipoth.
192	108	6
2304	1296	000
1296		12
3600		

2. When you have the Hipothenuſe and one of the Legs given, to find the length of the other Leg.

You muſt ſquare the lengths of the Hipothenuſe and the other Leg ſeverally, and Subſtract the leſſer Square from the greater, and from the Remains extract the Square Root, and that Root will be the length of the other Leg.

Example,

In the ſame Triangle, ſuppoſe the length of the Hipothenuſe to be 60 Lea. and the given Leg to be 36 Lea. the length of the other Leg is required.

60 Lea.	36 Lea.	2304. (48 Lea. the other Leg.
60.	36.	4
3600	216	704
1296	108	88
2304	1296	00

The preceeding Eight Caſes of right Lin'd Triangles are performed by natural Arithmatick, without the help of Tables.

When

When you have the Angles of a right Angled right Lin'd Triangle, add one of the sides given, you must always suppose another Triangle, whose Leg opposite to the lesser Angle must always be 1.00, and the following Rule will give you the length of the other Leg and the Hipothenuse.

The general Number given to Work by is 172, which you must divide by the lesser of the two given Angles, which being opposite to the lesser Leg of the Triangle given, which Leg in the assum'd Triangle (must always be supposed to be 1.00) square the Quotient, and from that Product Substract 3, and from the Remains extract the square Root, then double the Quotient, and from that Sum Substract the square Root, the remains being divided by 3, the Quotient will be the length of the Hipothenuse in the assum'd Triangle, which assum'd Hipothenuse being doubled, Substract that Sum from the Quotient of the first Division, the remains will be the greater Leg of the assum'd Triangle.

Note, That for the distinction in this Work, I mark the proposed Triangle with the Capital Letters A B C, and the assum'd Triangle with the small Letters a b c.

Therefore when the Solution of any right Angled right Lin'd Triangle is proposed by this Method, having the Angles and one of the sides given whereby to find the other sides, you must assume another right Angled right Lin'd Triangle, which must have the same Angles a

the

the Triangle propoſed; for if you obſerve which ſide is given in the propoſed Triangle, the ſame ſide in the aſſum'd Triangle will be correſponding thereunto, comparing like ſides; then as any one ſide in the aſſum'd Triangle is to its correſponding ſide in the propoſed Triangle, ſo is any other ſide in the aſſum'd Triangle to its correſponding ſide in the propoſed Triangle, and comparing like Sides, every ſide in the propoſed Triangle may be exactly found, and this Method fails not to ſolve the Queſtion unto two or three places, but if the Operation be continued unto two or three places in the Decimals, the Solution will be exact enough for any uſe. This is clearly demonſtrated in the Sixth Book of the Elements of Geometry, that thoſe two Triangles, *viz.* that propoſed and that aſſum'd, their Angles being alike, are in Proportion the one to the other.

Note, That when the quantity of the Angles are given in Degrees and Minutes, they muſt be reduced to Decimal Parts.

Caſe the Firſt.

The Baſe and Angles given, to find the length of the Perpendicular and the Hipothenuſe.

Example, Plate 2. Fig. 20.

In the right Angled right Lin'd Triangle A*B* C, right Angled at B, there is given the Baſe A B 85 Lea. the Angles A C B 54 deg. 24.

or 54 deg. 4 pts. the Angle B A C 35 deg. 36, or 35 deg. 6 pts. to find the length of the Perpendicular B C and the Hipothenuſe A C.

B A C		Quotient 4. 83
d. pts.	Quot.	
35. 6) 172.000 (—4.83		doubled. 9.66
	4.83	Subſtr. □ R 4.5
2960		
	1449	Rem.dv.by 3) 5.16
1120	3864	
	1932	aſſum'd Hip. a c. 172
52		double it and
Quotient □ d.	23.3289	ſub. from quo. 3.44
Subſtract	3.	aſſum'd Leg a b. 1.39
Extract the	20.3289	□ R
	4	4. 5.
	432	
	85	
	789	

1. To find the length of the Hipothenuſe A C.

as Aſſ. Leg. a b 1.39 A C.. Leg. A B 85 Lea:: Aſſ. Hip. ac 1. 72. Hip. 105 Lea.

1. 39.) 146.20. (105. Lea. A C.

	85
720	860
	1376
25	146.20.

2. To

2. To find the length of the Leg B C.
As the Aſſ. Leg. a b 1. 39 .. Leg. A B 85 :: Aſſ. Leg. b c 1.00. Leg. B C 61.1 Lea.

100

139) 85.000 (61. 1 Lea. B C 8500

160

210

71

Caſe the Second.

The Hipothenuſe and Angles given, to find the length of the Baſe and the Perpendicular.

Example, Plate 2. Fig: 21.

In the right Angled right Lin'd Triangle A B C, right Angled at B, there is given the Hipothenuſe A C 99 Lea, the Angle BAC 31 deg. 30, or 31 deg. 5 pts. and A B C 58 deg. 30, or 58 deg. 5 pts. to find the length of the Baſe AB, and the perpendicular B C.

B A C Quot.

31. 5) 172.000 (— 5.46
1450 546
1900 3276
2184
10 2730

Quotient □ d. 29.8116
Substract — — 3— - □ ℞
Extract the 26.8116 (5 17
5
181
101
8016
1027
827

Quot. 5.46
doubled 10.92
subst. the □ ℞ 5.17
Rem. div. by 3) 5.75
Ass. Hip. a c 1.91
double it and
sub. from quo. 3.82
Ass. Leg. a b 1.64

1. To find the length of the Base A B.
As Ass. Hip. a c 1 91. Hip. A C 99 : : Ass. Leg. a b 1. 64 .. Leg. A B 85. Lea:

99.

1. 91.) 162.36 (85 Lea. A B 1476
956 1476
01 16236

2. To

2. To find the length of the Perpendicular B C.

As Aſſ. Hip. ac 1.91 .. Hip. A C 99 :: Aſſ. Leg. b c 1.00 Leg. B C 51 Lea.

```
                                        1.00
                                       -----
1.91) 99.00 (51 Lea. B C.              99.00
      -----
        350
        ---
        159
```

Caſe the Third.

The length of both the Legs being given, to find the length of the Hipothenuſe and the quantity of the Angles.

1 *Note*, That before you can find the quantity of the Angles, you muſt find the length of the Hipothenuſe by the extraction of the ſquare Root, as before directed.

2 *Note*, When you have found the length of the Hipothenuſe, then you have the three ſides of a right Angled right Lin'd Triangle given, to find the Angles, and when you would find either of the Angles you muſt make uſe of the common Number 86 deg. in the ſtead of Radius (or 90 deg.) and then ſay.

As the length of the Hipothenuſe and half the longeſt Leg is to 86 deg. (the common Number) ſo is the length of the ſhorteſt of the two Legs to the quantity of its oppoſite Angle in Degrees and Decimal Parts.

Example, Plate 2. Fig. 22.

In the right Angled right Lin'd Triangle A B C, right Angled at B, there is given the length of the Base 71 Lea. and the Perpendicular B C 63 Lea. to find the length of the Hipothenuse A C, and the quantity of the two Angles B A C and A C B.

1. To find the length of the Hipothenuse A C, by the Extraction of the square Root.

```
AB 71 Lea.   BC 63 Lea.   9010.00( □ ℞
                                   94. 9 A C.
     71           63       9
   ----         ----     ----
     71          189      910
   497          378       184
   ----         ----     ----
  5041         3969.    17400
  39.69                  1889
  -----                  ----
  9010.                   399
```

2. To find the quantity of the Angle BAC.

```
            A B  71
               ----
      Half A B 35.5
           A C 94.9
               ----
```

As A C and half A B 130.4 .. 86 deg :: B C 63 .. B A C 41 deg. 5 pts. which subst.

```
                                      63
              d.pts.                 ----
130.4) 5418.00 41 5 BAC  258  leaves BAC 41.5
        2020                516         from 90.0
        ----               ----             ----
        7160               5418
        ----
         640                            ACB 48.5
```

Case the Fourth.

The length of Hipothenuse and one of the other Legs given, to find the other Leg and the quantity of the Angles.

Example, Plate 2. Fig. 23.

In the right Angled right Lin'd Triangle ABC, right Angled at B, there is given the length of the Hipothenuse AC 83 Lea. and the length of the Base AB 74 Lea. to find the length of the Perpendicular BC, and the quantity of the Angles BAC and ACB.

1. To find the length of the Perpendicular BC, by the Square Root.

AC 83. Lea.	AB 74 Lea.	1413.00(37.5 Lea. BC
83	74	3
249	296	513
664	518	67
6889	5476	4400
5476		745
1413		575

2: To find the quantity of the Angle *BA*C.

AB 74

Half A*B* 37
AC 83

As

As A C and half A B 120 .. 86 deg :: B C 37.
5 :: Angle B A C 26 deg. 87 pts.

	86
	2250
	3000
120.) 3225.00 (26 deg. 87 pts. B A C	3225.0
825	
1050	
9000	
60	

Now the Angle B A C is found to be 26 deg. 87 pts. which being Substracted from 90 deg. will leave 63 deg. 13 pts. the quantity of the Angle A C B.

Of oblique right Lin'd Trigonometry.

IN the three first Cases wherein there is an Angle given, they may be resolved by the preceeding Rules of right Angled right Lin'd Triangles, and to perform the same you must reduce the oblique right Lin'd Triangle, into two right Angled Triangles by letting fall a Perpendicular, which must always be from the Extream of a given side, and opposite to a given Angle adjacent to the given side, and then the given side will be the Hipothenuse and the given Angle; the Angle at the Base, and the Triangle wherein the Side and Angle is given may be called the First Triangle, and the other the Second.

Ca

Case the First, or Fifth.

Two Sides with an Angle adjacent to one of them given, to find the Third Side and the other two Angles.

Note, That in all Questions partaining to this Case, the Perpendicular must always fall upon a required Side, therefore if the longest side be required it will fall within the Triangle, but if either of the lesser sides is required it will fall without the given Triangle, and the side upon which it is to fall must be extended, and for the Solution thereof (it being divided into two right Angled Triangles) in the first, there is the Hipothenuse and Angles known whereby to find the Legs, and in the second right Angled Triangle, there will be the Hipothenuse and one of the Legs, known whereby to find the other Leg and Angles.

Example, Plate 3. Fig. 29.

In the oblique right Lin'd Triangle A B C, there is given the Side A C 56 Lea. the side B C 42 Lea. and the Angle B A C 29 deg. 5 pts. to find the quantity of the other two Angles A B C and A C B, and the length of the third side A *B*.

First, You must let fall a Perpendicular from C, as C D, which will divide the oblique Triangle A B C into two right Angled Triangles, as A C D and *B* C D.

In the first right Angled Triangle A C D, there is given the Hipothenuse A C 56 Lea. and the

the Angles D A C 29 deg. 30. and A C D 60 deg. 30, to find the length of the two Legs A D and C D.

	Quot.	Quotient 5. 83
29. 5) 172.000	(—5.83	doubled. 11.66
	5.83	Subſtr. □ ℞ 5.56
2450		
	1749	Rem.dv.by 3) 6.10
900	4664	
	2915	aſſum'd Hip. a c. 203
15		double it and
Quotient □ d.	33.9889	ſub. from quo. 4.06
Subſtract	3.	aſſum'd Leg a b. 1.77
Extract the	30.9889	(□ ℞ 5. 56.
	5	
	598	
	105	
	3789	
	1106	
	753	

1. To find the length of the Leg A D:
As Aſſ. Hip. a c 203 .. Hip. A C 56 :: Aſſ. Leg. a d 1.77. Leg 48. 8 Lea.

	1.77
	56
	1062
2.03)99.120 (48 8 Lea. AD	885
1792	99.22
1680	
56	

2. To find the length of the Leg CD.
As Aſſ. Hip. a c 2. 03.. Hip. A C 56 :: Aſſ. Leg. c d 1.00 Leg CD.

	1.00
2.03) 56.000 (27. 5 Lea. C D	5.600
1540	
1190	
185	

Then in the ſecond right Angled BCD, there is given the Hipothenuſe B C 42 Lea. and the Leg C D 27. 5 Lea. (that was found) to find the Leg B D, and the Angles B C D and C B D.

1. To find the length of the Leg B D, by the Square Root.

CB 42 Lea. C D 27.5 Lea. □ R

42	27.5	1007.25 (31. 7 Lea. BD
84	1375	3
168	1925	107
1764	550	61
756.25	756.25	4625
1007 75		627
		236

2. To find the quantity of the Angle CBD.

B D 31. 7 Lea.

Half B D 15.85

B C 42.

As BC & ½ *BD* ——

57.85 :: 86 :: C D 27.5 :

	86
57.85) 2365.000 (40d. 8 CBD	1650
.31000	2000
4700	2365.0

	d. p:
	40.8
Being sub: from	90.0
The Angle BCD	49.2
Then Add ACD	60.5
It makes ABC	109.7

And if you add the Leg AD 48 . 8 Lea:
Unto the Leg B D ——— 31 . 7

It makes the side AB ——— 80 . 5, as required.

Example

Example 2. *Case the Second. Plate* 3. *Fig.* 30.

In the oblique right Lin'd Triangle A B C there is given the side A B 80 5 Lea. the side B C 42 Lea. the Angle A C B 109 deg. 7 pts. to find the length of the side A C, and the quantity of the Angles A B C and B A C.

For, to reduce this oblique Triangle, into two right Angled Triangles, you must extend the required side A C, and from B the extream of the side *B* C, let fall a Perpendicular to cut the side B C extended in D, and you will reduce the oblique Triangle A B C into two two right Angled Triangles, B C D and *A* B D.

In the first Triangle B C D you have the Hipothenuse B C 42 Lea. and the Angle BCD 70 deg. 3 pts. found by Substracting the obtuse Angle A C B 109 deg. 7 pts. from 180 deg. leaves 70 deg. 3 pts. which being Substracted from 90 deg. will leave 19 deg. 7 pts. the quantity of the Angle C B D, whereby to find the length of the Legs B D and C D.

	Quot.	Quot.	8.73
19.7) 172.000 (	8.73		
1440	8.73	doubled	17.46
		subst. the □ ℞	8.55
.610	2619		
	6111	Rem. div. by 3)	8.91
19	6984		
		Ass. Hip. b c	2.97

Quot. □ d. 76.2129
Substract — 3 — — □ ℞ double it and
Extract the 73.2129 (8.55 sub. from quo. 5.94
8

Ass. Leg. b d 2.79

921
165

.9629
1705

1104

1. To find the length of the Leg BD.
As Ass. Hip. b c 2.97. Hip. BC 42 :: Ass. Leg. b d 2.79 .. Leg. BD 39.4 Lea:

2.79
42.

2.97.) 117.180 (39.4 Lea. BD   558
1116

2808
—
11718

1350
162

2. To find the length of the Leg C D:
As Aſſ. Hip. b c 2.97 .. Hip. B C 42 :: Aſſ. Leg. c d 1.00. C D 14. 1 Lea.

1.00

42.00

And by the preceeding Operation you have another right Angled Triangle as A B D, wherein you have the Hipothenuſe A B 80.5 Lea. and the Leg B D 39.4 Lea. to find the length of the Leg A D, and the quantity of the Angles B A D and A B D.

1. To find the length of the Leg A B by the Square Root.

AB 80.5 Lea. BD 39.4Lea. 4927.89 (□ ℞ 70.1Le:AD

80.5	39.4	7
4025	1576	02789
64400	3546	1401
	1182	
6480.25		1388
1552.36	1552.36	
4927.89		

2. To find the quantity of the Angle BAD.

A D 70.1 Lea:
½ A D 35.05
A B 80.5

As AB & ½ AD— 115.55.. 86 d :: BD 39.4:. BAD 29. 3 d, being ſub from 90. 0

115.55) 3388.400 (29 d: 3 pts.	86	from 90. 0
107740	2364	leaves 60.7 ABD
37450	3152	
2789	3388.4	

Now

Now for to find the length of the third side A C of the oblique Triangle A B C, you must Substract the Leg C D 14.1 Lea. from the Leg A D 70.1 Lea. and it will leave A C 56 Lea. and to find the quantity of the Angle A B C you must take the quantity of the Angle C B D 19 deg. 7 pts. from the quantity of the Angle A B D 60 deg. 7 pts. and it will leave 41 deg. 0 pts. the quantity of the Angle ABC.

Case the Second, or Sixth.

The Angles of an oblique right Lin'd Triangle with one of the sides given, to find the other two sides.

Note, That in all Questions pertaining to this Case, the Perpendicular according to the preceeding Rules, must always fall upon a required side, for if either of the lesser sides be given, it will fall without the Triangle and upon one of the required sides being extended, and for the Solution thereof, in the first right Angled Triangle there is the Hipothenuse and Angles given to find the Legs, and in the second Triangle you will have the Angles and one of the Legs, known whereby to find the Hipothenuse and the other Leg.

Example, Plate 3. Fig. 31.

In the oblique right Lin'd Triangle A B C, there is given the side A C 75 Lea. the obtuse Angle A C B 115 deg. 4 pts. and the Angle B A C 28 deg. 5 pts. to find the length of the two sides A B and B C.

In

In the first right Angled Triangle, there is iven the Hipothenuse A C 75 Lea. and the two ngles B A C 28 deg. 5 pts. and A C D 61 deg. pts. to find the length of the two *Legs* A D nd C D.

AD		Quotient	6. 03
	Quot.		
8. 5) 172.000	(—6.03	doubled.	12.06
	6.03	Substr. □ ℞	5.77
1000			
	1809	Rem.dv.by 3)	6.29
145	36180		
		assum'd Hip. a d.	2.09
Quotient □ d.	36.3609	double it and	
ubstract	3.	sub. from quo.	4.18
		assum'd Leg a d.	1.85
xtract the	33.3609	(□ ℞	
	5	5. 77.	
	836		
	107		
	8709		
	1147		
	580		

1. To

1. To find the length of the Leg A D.

As Aſſ. Hip. a c 2.09. Hip. A C 75 : : Aſſ. Leg. a d 1.85..Leg AD 66.3 Lea.

2:09) 138.750 (66.3 Lea. A D.	1.85
	75
1335	
	925
.810	1295
183	138.75

2. To find the length of the Leg C D.

As Aſſ. Hip. a c 2.09.. Hip. A C 75 : : Aſſ. Leg c d 1.00.. Leg C D.

	1.00
1.09) 75.000 (35.8 Lea. C D	75.00
1230	
1850	
178	

In the ſecond right Angled Triangle C B D there is given the Leg C D 35.8 Lea. and the Angles B C D 53 deg. 9 pts. and C B D 36 deg. 1 pts. to find the length of the Hipothenuſe B C and of the Leg B D.

☞ The quantity of the Angle B C D, is found by Snbſtracting the quantity of the obtuſe Angle A C D 115 4 pts. will leave the quantity of the Angle B C D 53 deg. 9 pts.

C B D

CBD

```
              Quot.
36.1) 172.000 ( 4.76
        ————    4.76
      2760    ————
        ————    2856
      2330    3332
       ———    1904
      174———
Quot. □ d.    22.6576
Substract —   3 —-   □ R
Extract the   19.6576(4.43
              4
            ————
              365
               84
             ————
              2976
               883
             ————
              327
```

```
               Quot. 4.76
                     ————
             doubled 9.52
     subst. the □ R  4.43
                     ————
   Rem.div.by 3)5.09
        Ass.Hip. cb  1.69
                     ————
      double it and
     sub. from quo.  3.38
                     ————
       Ass. Leg. cd  1.38
```

1. To find the length of the Hipothenuse BC
As Ass.Leg.cd. 1.00.. Leg.CD 36.1 :: Ass. Hip. cb 1.69 .. Hip. BC 61 Lea.

```
      1.69
     ——————
     3249
    2166
    361
   ——————
   61.009 Lea : B C .
```

2. To find the length of the Leg B D.

As Ass. Leg. c d 1.00.. Leg. C D 36.1 :: Ass. Leg b d 1.38 Leg B D. 49.8 Lea

	36.1
	1.38
	2888
	1083
	361
	49.818 Lea. B D

Example 2. *Case the Sixth. Plate* 3. *Fig*. 32.

In the oblique right Lin'd Triangle A B C there is given the side A B 116.7 Lea. the Angle A C B 115 deg. 4 pts. and the Angle B A C 28 deg. 5 pts. to find the length of the two sides A C and B C.

According to the preceeding Rules, the perpendicular will fall without the Triangle, either from B upon the side A C extended, or from A to the side B C extended. Here the perpendicular is let fall from B upon the side A C extended, dividing it into two right Angled Triangles, as A B D and B C D.

In the first right Angled Triangle A B D, you have given the Hipothenuse AB 116.7 Lea. and the Angles B A B, 28 deg. 5 pts. and the Angle ABD 61. deg. 5 pts. to find the length of the two Legs A D and B D.

B A

BAD Quot. 6.03

28.5) 172.000	Quot. (—6.03	doubl.	12.06
	6.03	Subst. □ ℞	5.77
1000	1809	Rem.div. by 3)	6.29
145	36180	Ass. Hip. a b.	2.09
Quot. □ d.	36.3609	double it and sub. from the Q.	4.18
Substract	3	ass. Leg a d.	1.85
Extract the	33.3609	(□ ℞ 5. 77.	
	5		
	836		
	107		
	8709		
	1147		
	680		

1. To find the length of the Leg A D.

As Ass. Hip. a b 2.09. Hip. A B 116.7 :: Ass. Leg. a d 1.85..Leg A D 103.2 Lea: 1.85

2.00) 215.895	103.2 Lea. A D.	5.835
689		9336
625		1167
207		215.895

2. To

2. To find the length of the Leg B D:

As Ass. Hip. a b 2.09. Hip. A B 116.7 :: Ass. Leg. b d 1.00. Leg. B D 558 Lea.

1.00

2.09) 116.700 (55.8 Lea. B D   116.700

1220

1750

78

In the second right Angled Triangle, B C D you have the Leg B D 55.8 Lea. and the Angles B C D 64 d. 6 pts. and C B D 25 d. 4 pts. to find length of the other Leg C D, and the Hipothenuse B C.

CBD   Quotient

25.4) 172.000 (—6.77

6.77

1960

4739

1820

4739

4062

.42

Quotient □ d. 45.8329

Substract— 3 □ R.

Extract the 42.8329 (6.54

6

6 83

1 25

. 5829

1304

613

Quotient ——— 6.77

Doubled——— 13.54

Substract the □ R. 6.54

Rem. Divide by 3) 7.00

Ass. Hip: b c —2.23

Double it and sub. from first Quot. } 4.66

Ass. Leg. b d ——— 2.11

1. To find the length of the Hipothenuse BC.
As Aſſ. Leg b d 2.11 .. Leg B D 55.8 :: Aſſ.
Hip. 2.33 .. Hip. BC 61.1 Lea.

```
                                  2.33
                                 -----
2.11)130.014(61.1 Lea. BC         1674
     -------                     1674
      .341                      1116
                                ------
     ------                     130.014
      .304
     ------
      .93
```

2. To find the length of the Leg C D.
As Aſſ. Leg b d 2.11 .. Leg. B D 55.8 :: Aſſ. Leg.
c d 1.00.. Leg. C D 26.4 Lea.

```
                                1.00
                               ------
                               55.800

2.11) 55.800 (26.4 Lea. C D
      ------
       1360
      ------
        .940
      -------
        .96
```

Then if you ſubſtract the length of the Leg
CD 26.4 Lea. from the Leg B D 103.2 Lea. it
will leave the length of the Side A C 76.8 Lea.
as was required.

Caſe

Case the Third, or Seventh.

Two sides with their contain'd Angle given, to find the other two Angles and third side.

In all Questions under the consideration of this Case, the Perpendicular may fall either within or without the Triangle (according to the preceeding Rules) but it must fall upon one of the given sides, this is when the given Angle is Accute. But if the given Angle is Obtuse, it will always fall without the Triangle, but still upon one of the given sides Extended.

Example, Plate 3. Fig. 33.

In the Oblique right Lin'd Triangle A B C. there is given, the side A B 357 Lea. the side A C 273 Lea. and the contain'd Angle B A C 33 deg. 6 pts. to find the quantity of the other two Angles ACB and A B C. and the length of the side B C.

To reduce this Oblique Triange into two right Angled Triangles, you must let fall a Perpendicular from C upon the side A B. which will reduce it into two right Angles, as ACD and BCD.

In the first right Angled Triangle A C D, there is given the Hipothenuse A C 27.3 Lea. and the Angles ACD 56 deg. 4 pts. and C A D 33 deg. 6 pts. to find the length of the two Legs A D and C D.

CAD

CAD

33.6)172.000	Quot. (-- 5.11	Quot. 5.11
.400	5.11	doubled 10.22
640	511	Subst. the □ ℞. 4.80
304	511	Rem. div. by 3) 5.42
	2555	Assum. Hip. a c 1.80
Quot. □ d.	26.1121	double it, and sub. from first q. 3.60
Substract	3	Assu. Leg a d 1.51
Extract the	23.1121	(□ ℞ 4.80
	4	
	711	
	88	
	.721	
	960	

1. To find the length of the Leg A D.

As Ass. Hip. a c 1.80 .. Hip. A C 273 :: Ass. Leg a d 1.51 .. Leg AD 228.9 Lea.

1.8)412.03 (228.9 Lea. AD
.52
160
163
.1

1.51
273
1365
273
412.03

2. To

2. To find the length of the Leg C D.

As Aſſ. Hip. a c 1.80. Hip. A C 273 :: Aſſ. Leg. c d 1.00. Leg C D 151. 1 Lea. 100

```
                   273.00
1.8) 273.00 (151.1 Lea. CD
      .93
       .20
        .20
        12.
```

Then if you ſubſtract the length of the Leg A D 228.9 Lea. from the Side A B 357 Lea. it will leave 128.1 Lea. the length of the Leg B D.

In the Second Triangle BCD, you have the length of the two Legs B D 151.1 Lea. and CD 128.1 Lea. given, to find the quantity of the Angles BCD and CBD, and the length of the Hipothenuſe B C.

1. To find the length of the Hipothenuſe BC, by the Square Root.

```
                                  Lea.
BD 151.1 L. CD 128.1 L. 39240.82(198 Hip.BC
   151.1       128.1     1
   -----       -----     ----
    1511        1281     292
   1511        10248      29
  7555         2562      ----
 1511         1281       3140
 --------    --------     388
 22831.21    16409.61    ----
 16409.61                 3682
 --------                 3960
 39240.82
```

2. To

2. To find the quantity of the Angle BCD.

BD 151.1 Lea.

½ BD 75.55
BC 198.

d   d.pt
As ½ BD & BC 273.55 .. 86 :: CD 128.1 .. BCD —— 40.2
86.   being sub. from 90.0

7686   leaves Ang. CBD 49.8
10248

11016.6

d p
273.55) 11016.600 (40.2 BCD
.074600

19950

Example 2. *Case the Third. Plate* 3. *Fig.* 34.

In the oblique right Lin'd Triangle ABC, there is given the side AC 273 Lea. the side *B*C 198 Lea. and the obtuse Angle ACB 97 deg. 1 pt. to find the quantity of the two Angles BAC & ABC, and the length of the side AB.

In this Triangle the perpendicular will fall without the Triangle, therefore extend the side AC, and let fall a perpendicular from B, to cut AC extended in D, and if you substract the given Angle ACB 97 d. 1 pt. from 180 d. it will leave BCD 82 d. 9 pts.

So that in the first right Angled Triangle BCD, you have the length of the Hipothenuse BC 194 Lea. and the quantity of the Angles BCD 82 d. 9 pts. and CBD 7 d. 1 pt. to find the length of the two Legs BD and CD.

CBD is

Quot. 24.22
doubled 48.44
Subst. the □ ℞. 24.15
Rem. div. by 3) 24.29
Assum. Hip. a c 8.09
double it, and
sub. from first q. 16.18
Assu. Leg b d 8.04

```
              Quot.
7.1 ) 172.000  ( 24.22
                 24.22
  .300          -----
                 4844
  160           4844
               9688
  180         4844
  38          --------
Quot. □ d.   586.6084
Substract       3
             --------
Extract the  583.6084   ( □ ℞
               2          24.15
             -----
              183
               44
             -----
              .760
               581
             -----
             17984
              4825
             -----
              3859
```

1. To find the length of the Leg B D.

As Ass. Hip. b c 8.09 .. Hip. B C 194 :: Ass. Leg b d 8.04 .. Leg BD 192.8 Lea.

```
                                        8.04
                                        ----
8.09) 1559.760 (192.8 Lea. BD           774
      7507                            15520
      ----                           -------
       2266                          1559.74
       6480
         16
```

2. To

2. To find the length of the Leg C D.

As Aſſ. Hip. b c 8.09.. Hip. B C 194 :: Aſſ. Leg c d 1.00.. Leg CD 23.9 Lea.

```
                              1.00
                              ----
                             19400
8.09 ) 194 000 (  23.9 Lea. C D.
        3220   ( 273.0 Lea. AC being added,
         7930  (296.9 Lea. AD
         ----
          649
```

And in the ſecond right angled Triangle ABD you have the Leg A B 296.9 Lea. and the Leg. BD 192.8 Lea. given, to find the length of the Hipothenuſe A B, and the quantity of the Angle B A D.

1. To find the length of the Hipothenuſe AB, by the Square Root.

```
                                    . . . . Lea.
AD 296.9 L.  BD 192.8L.   125321.45 (354. AB.
   296.9        192.8     3
   -----        -----     ----
    26721        15424    353
   17814        3856      65
  26721       17352       ----
  5938        1938        2821
  ---------   -----        704
  88149.61    37171.84    ----
   37171.84                0545
  ---------                7080
  125321.45
```

2. To find the quantity of the Angle BAD,

A D 296.9 Lea.

$\frac{1}{2}$ A D 148.45
A B 354.

As $\frac{1}{2}$ A D & A B 502.45 .. 86 d. :: B D 192:8:. B A D —— 3 d. 2. pt

86.

11568
15424

16580.8

502.45) 16580.800 (32.9 d. pts. B A D
150730
502400
50195

Case the Fourth, or Eighth.

The three ſides of an oblique right lin'd Triangle given, to find either of the Angles.

Example, Plate 3. Fig. 35.

In the Oblique right Lin'd Triangle A B C, there is given; the ſide A B 584 Lea. the ſide A C 398 Lea. and the ſide B C 268 Lea. to find the quantity of the Angle A C B.

In this Triangle I make A B the true Baſe, and therefore the Perpendicular muſt be let fall from the Angle C, upon the Baſe A B, and within the Triangle.

1. To

1. To find the alternate Base A D.

A C 398. Lea.
B C 268. Lea.

Their Sum 666. Lea.

Their Difference 130. Lea.

As true Base AB 584 .. Sum of AC & BC 666 :: their Diff. 130 .. alternate Base A D.

```
      666
      130
    -----
    19980
    666
    -----
    86580
```

```
584.) 86580.0 (148.2 Lea. A D
       2818
       4820
        1480
         312
```

Substract the alternate Base A D 148.2 Lea. from the true Base A B 584 Lea. it will leave BD 435.8 Lea. the half of which will be B 217 9 Lea. unto which if you add the alternate Base A D 148.2 Lea. it will make the Leg A E 366.1 Lea. and also you have the Hipothenuse A C 398. Lea. given in the first right angled Triangle A C E, to find the length of the Perpendicular C E, and the quantity of the Angles C A E and A C E.

1. To

1. To find the length of the Perpendicular, (or Leg) CE. by the Extraction of the Square Root.

	A C 398. Lea.	A E 366.1 Lea.
	398	266.1
	3184	3661
	3582	21966
	1194	21966
		10983
From it	158404.	
Substr.	134029.21	134029.21

24374.79 (155.9 Lea. C E.
1

143
25

1874
305

345·79
3109

6998

2. To

2. To find the quantity of the Angle C A E.

A E 366. 1 Lea:

½ A E 183. 05
A C 398.

As ½ AE & AC 581.05 .. 86 :: CE 155.9 .. CAE being 23. d.
86. & sub. from 90.

9354 leav. ACB 67
12472

13407.4

581.05) 13407.40 (23. d CAE

178640

.4325

In the second right angled Triangle B C E, you have given the length of the Perpendicular (or Leg) C E 155.9 Lea. the Leg BE 217.9 Lea. and the Hipothenuse B C 268 Lea. to find the quantity of the two Angles CBE & BCE.

1. To find the quantity of the Angle CBE.

BE 217.9 Lea:

½ BE 108.95
B C 268.

As ½ B E and BC 376.95 .. 86 d :: CE 155.9 . CBE being sub. 35.5 d. pt.
86. from 90.0

376.95) 13407.400 (35.5 CBE

9354 leaves BCE 54.5
12472 add ACE 67.0

2096 90

makes the requir'd Angle ACB 121.5

212150

13407.4

23675●

Example

Example 2. Case 8. Plate 3. Fig. 36.

In the oblique right lin'd Triangle ABC, there is given the side A B 584 Lea. the side A C 398 Lea. and the side B C 268 Lea. to find the quantity of the Angle A C B.

In this Example, I make the side B C the true Base, and therefore it must be extended, and then let fall a perpendicular from A, upon D, on the extended side B C.

First, You must find the alternate Base, whereby to reduce the oblique Triangle into two right angled Triangles.

A B 584. Lea.
A C 398. Lea.

their Sum 982. Lea.

their Difference 186 Lea.

As true Base B C 268 .. Sum of A B and A C 982 :: their Diff. 186.. alt. Base BE 681. 5 L.

```
  186
 ----
 5892
7856
 982
------
182652.
```

```
268.) 182652.0 ( 681.5 Lea. BE.
      ----
      2185
       .412
      ----
      1440
      ----
       100
```

Th

The Alternate Base B E ——————— 681.5 Lea.
The true Base B C substracted ——— 268. Lea.
The Remains will be C E ——— 413.5 Lea:
The half of which is C D ——— 206.75 Lea:
Unto which Add the true Base B C 268.
It will make B D ——— ——— 474.75

And doth Reduce the Oblique Triangle ABC into two Right Angled Triangles, as ACD and ABD.

In the first Right Angled Triangle ABD, you have the Hipothenuse A B 584 Lea: and the Leg B D 474.75 Lea: to find the length of the Leg A D: By the Extraction of the Square Root.

A B 584 Lea.
584
2336
4672
2920
341056
225387.5625
115668.4375

B D 474.75 Lea:
474.75
237375
332325
189900
332325
189900
2253875625

115668.43(340.1 Lea: DA
3
256
64
:06843
6801
..42

Then in the Second Right Angled Triangle ACD, you have the Lea. AC 398 Lea: and the two Legs, as AD 340.1 Lea: and CD 206.75 Lea. given to find the quantity of the Angle C A D.

AD 340.1 Lea.
½ A D 170.05
A C 398 —

As ½ A D and A C — 568.05 . . 86 :: C D 206.75 : C A D.
d. pts
86.

568.05)17780.500(31.3 C A D
·74100
186950
16545

124050
165400
17780.50

From ——— ——— ——— 90: 0
Substract the Angle C A D ——— 31. 3
The Remains will be the Angle ACD 58 . 7
Which being substracted from ——— 180: 0
Leaves the required Angle A C B ——— 121.8

Decimal Arithmetick.

BEfore you can proceed in Arithmetical Trigonometry, you ought to underſtand ſo much of Decimal Arithmetick, as to reduce Minutes into Decimal Parts, and for that Reaſon, I have given ſo much of Decimal Arithmetick as is neceſſary for the working thereof.

The Doctrine of Decimals.

This kind of Arithmetick is called Decimal or Mellicimal, becauſe all the Parts of an Integer is a Numerator, and muſt be in proportion unto a proper Denominator, which if it be but one Figure, is a Numerator to 10, if two Figures, to a 100, and if three Figures to a Thouſand, &c. The whole Numbers are diſtinguiſhed by a Point or Comma being placed between the whole Number and the Decimal Fraction, the whole Number ſtanding on the Left-hand ſide, and the Fraction on the Right-hand ſide.

The working of any Rule by Decimal Parts, taketh off the Trouble of the Reducing any Thing into the loweſt Denomination; for Fractions are as readily work'd jointly with whole Numbers as whole Numbers alone.

This ſort of Arithmetick is of excellent Uſe in the Menſuration of all Superficial and Sollid Meaſure; and in the working of Right Trigonometry Arithmetically.

Addition

Addition of Decimals.

This Rule in the Operation thereof, differeth nothing from Addition of whole Numbers in Vulgar Arithmetick, only you must observe to place the Units of the whole Numbers directly under each other, and likewise the Points or Comma's, and the same must be observed in the Total, or whole Sum.

Deg.	Pts.	L.	pts.	Feet.	pts.
57	51	136	71	36	172
94	374	391	567	43	53
314	05	47	47	171	075
76	723	65	368	59	68
5	67	136	44	64	294
548	327	777	555	374	751

In the working of Addition in Decimals, you begin at the Right-hand to cast them up, and for every Ten you must carry one to the next Row on the Left-hand, both in the Decimals and in the whole Numbers, taking no Notice of the Points or Comma's, until you have added them all together, and then place the Point or Comma in the Total, directly under the Points or Comma's of the Numbers that were added, and the Figures towards the Left-hand will be the whole Numbers, and those to the Right-hand the Decimal Parts.

Substraction of Decimals.

This for the manner of working is the same as the substraction of whole Numbers in Vulgar Arithmetick; only you must observe to place Units under Units, and the Points or Comma's as before in Addition.

Deg.	pts.	l.	pts.	Feet	pts.
567	· 07	430	· 5	543	· 05
374	· 186	234	· 673	74	· 237
192	· 884	195	· 827	468	· 813

The Proofs of Addition and Substraction of Decimals, is the same as in Vulgar Arithmetick.

Multiplication of Decimals.

THIS Rule, for the Method of Working, is the same as in Vulgar Arithmetick with this difference only, That you must always cut off so many Figures or Cyphers from the Product, towards the Right-hand, as there are Decimal Parts, both in the Multiplicand and the Multiplier.

But in the Multiplying of Decimal Parts only by Decimal Parts, the Product will be always less than either the Multiplicand or Multiplier, by which means it often happens that after the Multiplication is finished, there is not so many Figures and Ciphers in the Product, as there are Places in both the Decimal Fractions

ns, then you must always place so many Cy-
hers on the Left-hand side of the Product, as
ill supply that Defect.

☞ You Prove the Work as in Vulgar A-
thmetick.

57.64
34.8
———
46112
23056
17292
———
2005.872

67.85
.97
———
47495
61065
———
65.8145

35.7
.065
———
1785
2142
———
2.3205

579.
13.8
———
4632
1737
579
———
7990.2

.246
.053
———
738
1230
———
.013038

.754
46.
———
24
3016
———
34.684

.127
.234
———
508
381
254
———
.029718

.076
.093
———
228
684
———
.007068

.079
.0091
———
.0003711

.00976
.0084
———
3904
7808
———
.000081984

Division of Decimals.

THere being ſeveral Methods for the work-ing the Rule of Diviſion in Vulgar A-rithmetick, when you have placed the Divi-dend and Diviſor according to that ſort of Di-viſion you moſt affect, the Operation will b the ſame as in Vulgar Arithmetick, for all th difficulty that attends diviſion of Decimals mor than diviſion in vulgar Arithmetick is to fin the true value of the Quotient, that is, whe-ther it will be a whole Number, a mixt Num-ber, or a Fraction; and if a Mixt Number, t know where to place the Point or Comma whereby to diſtinguiſh between the whol Number and the Fraction; and for your grea-er help therein, obſerve the following gener Rule.

You muſt obſerve under what part of the D-vidend the Units Place of the Diviſor wi ſtand, for the firſt Figure in the Quotient wi always be of the ſame Degree or Place, as th Figure or Cypher in the Dividend is of, whi ſtandeth over the Place of Units in the Di-for: And you are further to take notice, th if neither the Dividend or Diviſor have Decimal Fraction belonging unto them, ye there ſhould be a Remainder after the Divi-on is finiſhed, you may annex Cyphers un the Dividend, until you have ſo many Pla in the Decimal Fraction as you require.

There are Eight ſeveral Queſtions in Di-ſion of Decimals.

1 To divide a whole Number, by a Fraction.
2 To divide a Fraction by a whole Number.
3 To divide a mixt Number by a Fraction.
4 To divide a Fraction by a mixt Number.
5 To divide a whole Number, by a mixt Numb.
6 To divide a mixt Number, by a whole Numb.
7 To divide a mixt Number, by a mixt Number.
8 To divide a Fraction by a Fraction.

The Division which I shall use in this Work, shall be that which is called the *Italian*, it being most used at this time.

It is required to divide 56. by .75, you must first place down the Dividend, and annex thereto so many Cyphers towards the Right-hand, as you shall think convenient, with a Point or Comma between the whole Number and the Cyphers; at each end of the Dividend draw a crooked Line, and on the Left-hand side place the Divisor thus, .75) 56.0000(

Then find how many times 75 you can have out of 56.0, and you will find it to be 7, place that 7 in the Quotient on the Right-hand side, and multiply 57 by 7, and having drawn a Line under the Dividend, substract the product of 75, being multiplied by 7, from 56.0 and place the Remains under the Line, and the Work will stand thus.

.75)56.0000(7
———
35

Then

Then take the next Figure or Cypher in the Dividend towards the Right-hand, and place it under the Line next to the Remains as it stands underneath.

.75)56.0000(7
350

Then find how many times 75 you can have in 350, and it will be 4 Times, which you must place in the Quotient, and having drawn another Line under the Remains, multiply 75 by 4, and substract the Product from 350, and placing the Remains under the last Line, and the Work will stand thus,

.75)56.0000(74.66
350
500

After the same manner you must proceed with the rest of the Dividend, until the Division is finished, now according to the general Rule, the first Figure in the Quotient will stand in the place of Tens, therefore from the Left-hand-side of the Quotient, count two Figures, and place a Point between the Second and Third Figures, and it is done.

You prove Division by multiplying the Quotient by the Divisor, and if there be any Remains belonging to the Division, add them to the Product, and if that Sum is equal to the Dividend, you are right, else not.

It is required to Divide .54321 by 16.

According to the General Rule, the Units place of the Divisor, should stand under the Second place of the Decimals in the Dividend, therefore you must place a Cypher before the Quotient, and a Point before that, and then the first Figure in the Quotient will be Seconds, and it will stand thus,

16.).54321 (.03395

.63

152

.81

81

☞ Note, That you must always place a Point under each Figure that you take to place under the Line, to know which Figure or Cypher you have taken down.

It is required to divide 7.8685 by .37

Here the Units Place of the Divisor should stand under the place of Tens in the Dividend, therefore the first Figure in the Quotient will be Tens, thus,

.37)7.8685(21.26

46

98

246

24

It is required to Divide .7594312 by 357.4.
Here the Units place of the Divisor stands under the place of Thirds in the Dividend, therefore you must place two Cyphers before the first Figure in the Quotient, as thus,

357.4)·7594312 (.002124

4463

8891

17432

3136

It is required to Divide 5678. by 37.56
Here the Units place of the Divisor standeth under the place of Hundreds in the Dividend, therefore the first Figure in the Quotient must be Hundreds, thus,

37.56)5678.0000(151.17

19220

4400

6440

26840

548

It is required to Divide 23·74567 by 47·5

Here the Units place of the Divisor standeth under the Primes of the Decimals, therefore the first Figure in the Quotient must be Primes, thus,

47·5) 23·74567 (·4999

4745

4706

4317

42

It is required to divide ·357359 by ·135

Here the place of Units in the Divisor, will stand directly under the place of Units in the Dividend, therefore the first Figure in the Quotient will be Units, as thus,

·135) ·357359 (2·647

873

635

959

14

How to find the Value of any Decimal Fraction.

YOU muſt multiply the Decimal Fraction, by a Number of the next lower Parts contained in its proper Integer, and the Product will be the Value of the Fraction in that Denomination, and the remaining Decimal, if any, are the Decimal parts of an Integer, it being of the ſame Denomination with the Multiplier, the value thereof may be found in the Inferior parts of the next Denomination, and ſo the leaſt parts of any Decimal Fraction may be found; for the Integer being known, the reſpective Products will ſhew the ſeveral parts of that Integer, as followeth.

☞ Note, That in the reducing of a Decimal Fraction, you muſt obſerve the general Rule for Multiplication of Decimals.

What is the Value of .67897 parts of a Foot in Inches? 12.

Inches 8.14764

What

What is the value of .89874 parts of a Pound Sterling in s. d. and qrs. 20.

s. 17.97480
12

d. 11.79760
4

qrs. 3.19040

What is the value of .789 parts of a Degree in Minutes? 60.

Minutes 47.340

The Rule of Three Direct in Decimals.

THis is the ſame as in Vulgar Arithmetick, only you always obſerve to place the Firſt and Third Numbers, ſo that they may be of one Denomination, (either of Money, Weight or Meaſure) for if the firſt Number be Money, the Third Number muſt be Money; and if the firſt Number be Weight or Meaſure, the third muſt always be the ſame.

For in the Rule of Three direct in Decimals, there is no occaſion for Reduction. But you may Multiply and Divide by either Fractions or mixt Numbers, without changing their Denominations.

l. pts: l pts. l. pts.
If 1 . 5 cost 1 . 765, what will 19. 25.

1.7 65
96 25
11550
13475
1925
33.97625

l. pts.
1.5) 33,97625(22.6508
39
97
76
125
5

l. pt. l. pt. l. pt.
For Proof, If 19 . 25 cost 22.6508 what will 1.5

1.5
1132545
226508
33.97625

l. pts.
19.25)33.97625(1.765
14726
12512
9625
0000

Note;

☞ Note, That in Multiplying the second and third Numbers into each other, I take in 5 the Remains of the first Division for exact Proof.

```
   s.p.  l. p.      s.   p.   l.    p.
If 2.5 .. 8,567 :: 437,75 .. 1500,9817.
                    8,567
                  -------
                  306425
                 262650
                218875
               350200
              ---------
              3750,20425

2,5)3750,20425( l.   pts.
    ---         1500,0817
    125
    ----
     00204
      ---
        42
       ---
        175
        ---
         00
```

Proof

```
    s.  p.   l.      pts.  l. pts. l. pts.
If 437,75 .. 1500,0817 :: 2,5 .. 8,567.
                   2.5
           ----------
          75004085  437,75)3750.20425(l. pts
        30001634           ---------- 8,567
        3750,20425          248204
                            ------
                             293292
                             ------
                              306425
                              ------
                               00000
```

If

s. p. l. p. s. p. l. p.
If 1,75 .. 2,5 :: 236.25 .. 337.5

```
     236.25
        2.5
    -------
     118125                               ( l. pts
     47250        1,75)590,625            ( 337.5
    -------              ----
    590,625              .656
                         ----
                          1312
                          ----
                           875
                           ---
                            00
```

S. pts. l. pts s. pts. s. pts.
Proof, If 236,25 .. 337,5 :: 1.75 .. 2,5

```
     337,5
     1, 75
    ------
     16875                                     ( l. pts
     23625        236,25)590,625               ( 2, 5
    3375                 -------
    ------               118125
    590,625              ------
                          00000
```

By what hath been done, you may see that the Rule of 3 in Decimals is performed in every respect as the Rule of 3 in whole Numbers, and more easily, but the Reader ought to have some Knowledge in Vulgar Arithmetick, if not, he ought to improve his Knowledge, before he enters upon Decimals.

THE

THE EXTRACTION OF THE SQUARE-ROOT.

WHEN you Extract the Square-Root of any Number proposed, you are to find out another Number, which being Multiplied into its self, the Product thereof will be equal to the Number first proposed.

For a Root is such a Number, that if Multiplied into its self, will produce another Number called a Square, as 3 times 3 is 9; here 3 is the Root and 9 is the Square, &c.

Square Numbers are either single or compound, single Square Numbers are such as are produced by the Multiplying of any one single Figure into its self, where the Product will always be less than a 100, as the following Table of Roots and Squares of single Numbers sheweth.

The Roots	1	2	3	4	5	6	7	8	9
Square Numbers	1	4	9	16	25	36	49	64	81

Compound Square Numbers are ſuch as are produced by the Multiplication of any Number conſiſting of more Places than one, and the Product never leſs than 100. The Root of any Number under 100, may eaſily be found at Sight by the Table of Single Squares, but for to Extract a Compound Number, and to find the Root thereof, obſerve the following Rules.

1. Place down the given Number, and ſet a Point over the Units Place, then miſſing every ſecond Figure towards the Left-hand, and place Points over the Third, Fifth and Seventh Figures, &c. and ſo many Points as you have over the given Number, of ſo many Figures will the Root conſiſt.

2. Then take the neareſt Root to the firſt Figure or Figures under that Point next to the Left-hand, draw a crooked Line on the Right Hand ſide of the given number, and place the Root found for a Quotient, multiply the Quotient (or Root) into its ſelf, and ſubſtract the Product from the Figure or Figures belonging to the firſt Point, (and having drawn a Line under the given Number) and place the Remains (if any) under that Line.

3 Then

3. Then bring down the Figures belonging to the next Point, and place them under the Line even with the Remains on the Right-hand ſide; this is called a Reſolvend, then double the Root and place the Units thereof under the Tens of the Reſolvend, this is called a Diviſor.

4. Then as in Diviſion, find how many Times you can find the Diviſor in the Figures over it, and place that Figure in the Quotient, and under the Units of the Reſolvend, and draw another Line under them, then Multiply all the Figures under the Reſolvend, by the Figure laſt placed in the Quotient, and ſubſtract the Product from the Reſolvend, placing the Remains under the laſt Line.

5. After the ſame manner you may proceed to any Number of Figures that ſhall be required in the Root, for after the firſt Figure is done, you muſt double all the Figures for a Diviſor, as if they were but one. To prove the Work, Multiply the Root into its ſelf, and take in the Remains, (if any) and if that Product is equal to the Number firſt propoſed, you are right, elſe not.

It is required to Extract the ſquare Root of } 1369

1 Having

1. Having placed Points over the given Number, draw a crookee Line on the Right-hand ſide for a Quotient, and it will ſtand thus.

```
 . .
1369(
```

2 Take the neareſt Root to 13. the Figures belonging to the firſt Point, which is 3, which place in the Root, as a Quotient, and work as in the ſecond Rule, and the Remains will be 4, and

```
1369(3
 3
-----
 4
```

3. Then bring down 69, the Figures belonging to the next point, and placing them to the Left-hand of the Remains, and it make 469, for a Reſolvend the Root 3 being doubled maketh 6, place the Units of that doubled Number, under the Tens of the Reſolvend, for a Diviſor as in the fourth Rule, and it ſtands thus,

```
1369(37
 3
-----
 469
  67
-----
  00
```

Note, That 1369 is an exact Compound ſquare Number, for the Points being all brought down, and after the laſt Subſtraction, there remains nothing, and 37 is the ſide or Root, which being Multiplied into it ſelf, will produce the propoſed ſquare Number 1369

The Square Root is Deman-ed of } 258064(508

Here you must obſerve, that vhen the Diviſor cannot be ta-en out of the Reſolvend, place Cypher in the *Quotient*, and ring down the Figures belong-ng to the next point, and place hem on the Right-hand ſide of he Reſolvend, for a new Reſol-vend.

```
258064(508
5
------
08064
1008
------
000
```

What is the Square root of --2267oooo(47.6

```
22670000(47.6
4
------
667
87
------
5800
946
------
124
```

But when all the points in the iven Number are finiſhed, if here ſhould be a Remainder af-er the laſt Subſtraction, they are alled Surd Numbers, being not Aenſurable to their Square roots, et may be obtain'd very near by he following Rule: When a Surd Number is propoſed, for Extraction, conſiſting of Integers only, you may Work according o the preceding Rules, but there will be a Remainder, which ſhews that the Number propoſed is Surd, and alſo that you have found he greateſt whole Number, that the propoſed Number will conſiſt of, but to find the Deci-mal Fraction belonging to ſuch a Root, for to bring it nearer the Truth, place two Cyphers upon the Right-ſide of the Remains for a new Reſolvend, then finding a new Diviſor as di-rected, and proceed in every reſpect according

to

to the Third and Fourth Rules, and you ma
produce another Figure for the Quotient, (a
will be the first Figure in the Decimal Fract
on,) which you must distinguish by a Point
Comma from the Integer, and so by a con
nual annexing two Cyphers unto every last r
mainder, you may continue the Extraction
so many Decimal Parts as you think fit.

What is the square Root of—13579246(36.8

3
457
66
6192
728
36846
7365
21

For if the Reader makes himself perfect
the preceding Rules of Decimal Arithmeti
and the Extraction of the Square Root, he m
be capable of performing any thing in Rig
lin'd Arithmetical Trigonometry.

FINIS.

Plate 1st

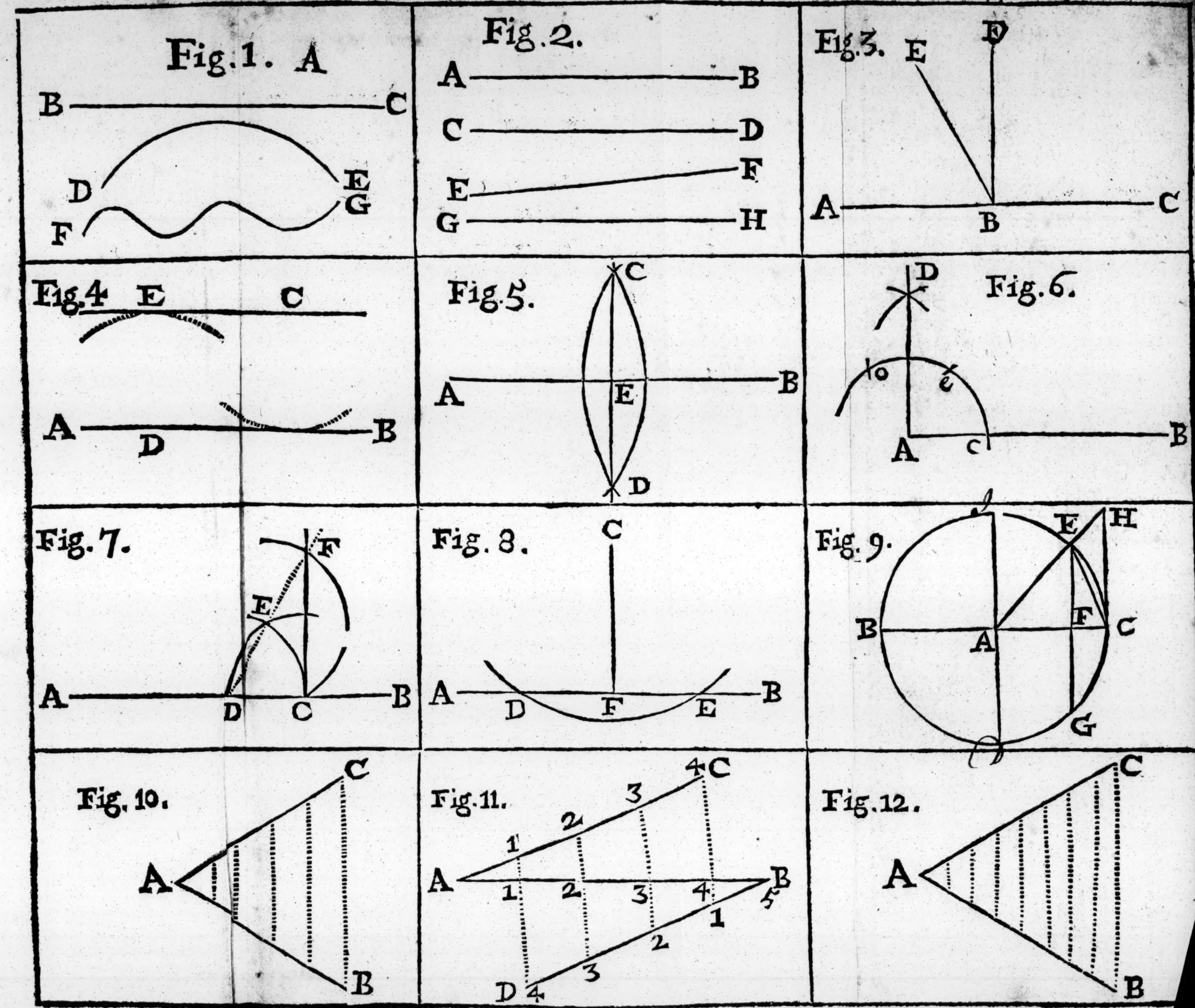
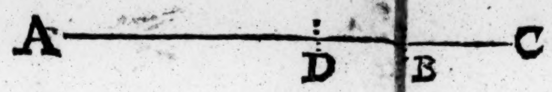


Fig. 1.
A
B
C
D
E
G
F
Fig. 2.
A
B
C
D
E
F
G
H
Fig. 3.
E
D
A
B
C
Fig. 4
E
C
A
D
B
Fig. 5.
C
A
E
B
D
Fig. 6.
D
o
e
A
c
B
Fig. 7.
F
E
A
D
C
B
Fig. 8.
C
A
D
F
E
B
Fig. 9.
d
E
H
B
A
F
C
G
D
Fig. 10.
C
A
B
Fig. 11.
4 C
3
2
1
A
1
2
3
4
B
5
1
2
3
D 4
Fig. 12.
C
A
B

Plate 2d

Fig. 13



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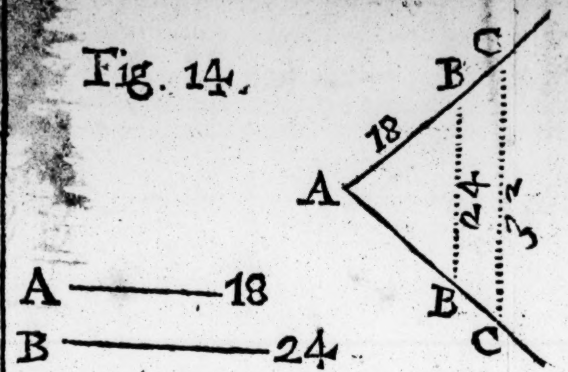

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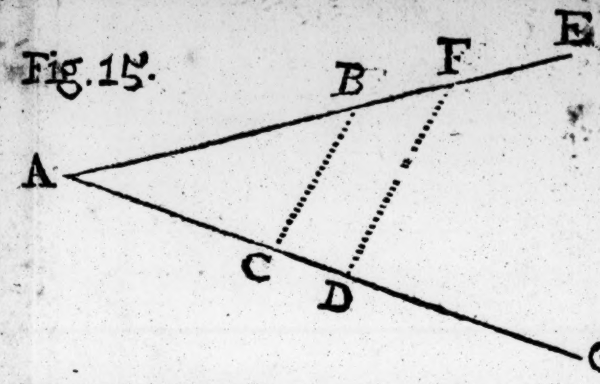

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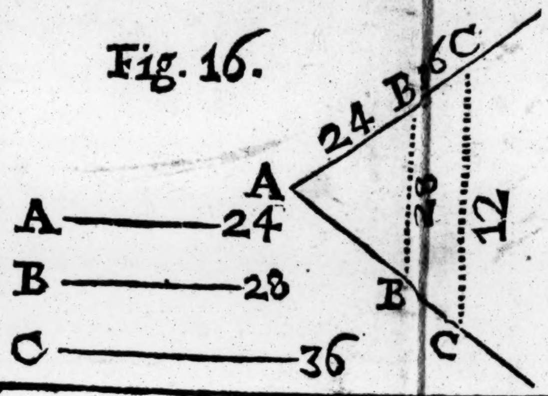

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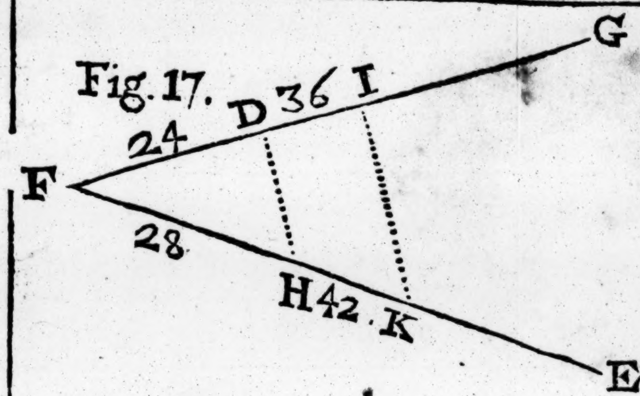

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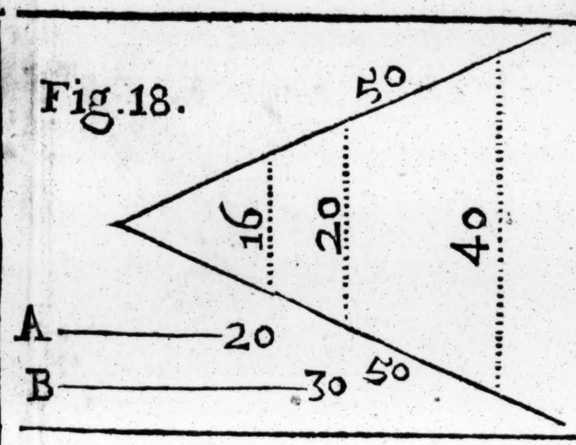

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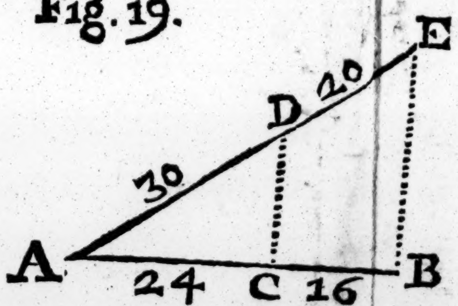

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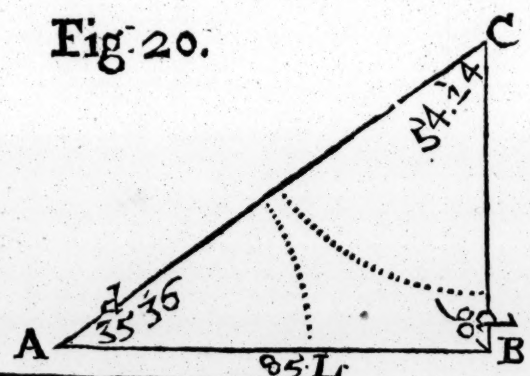

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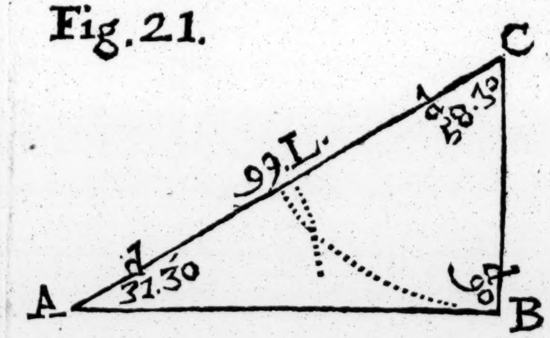

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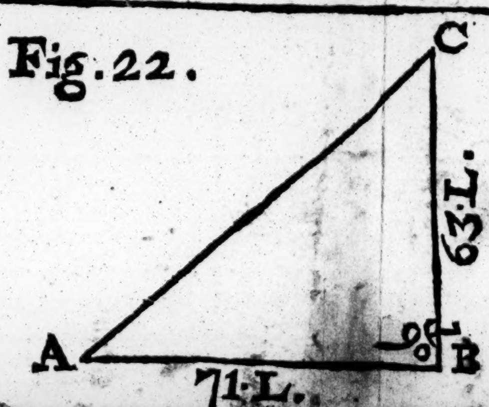

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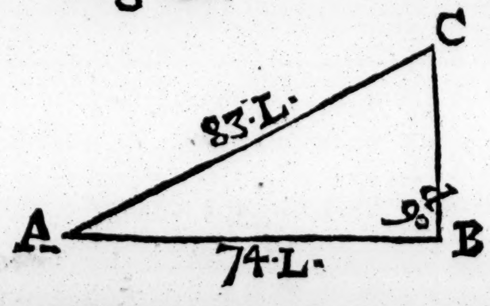
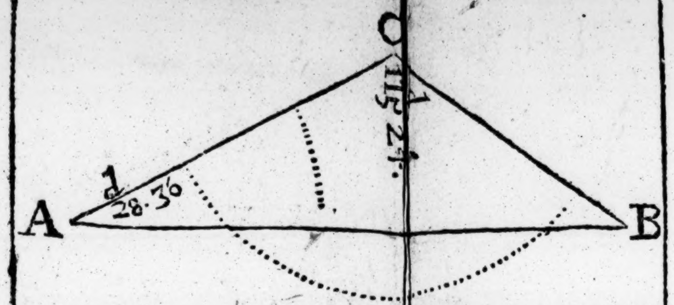


Fig. 24

Plate 3rd

Fig. 25.



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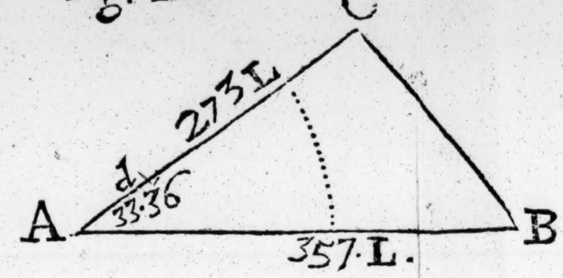

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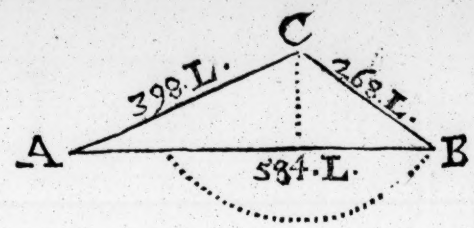

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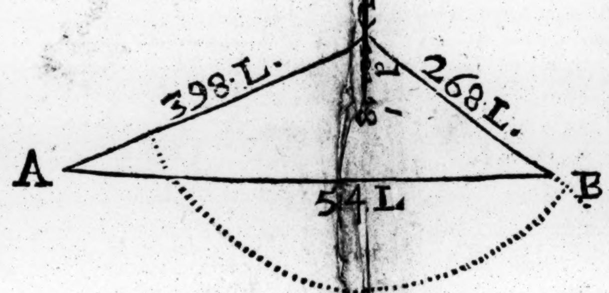

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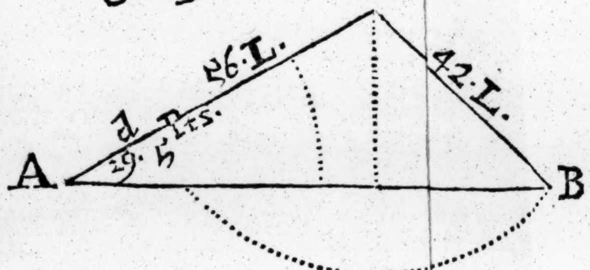
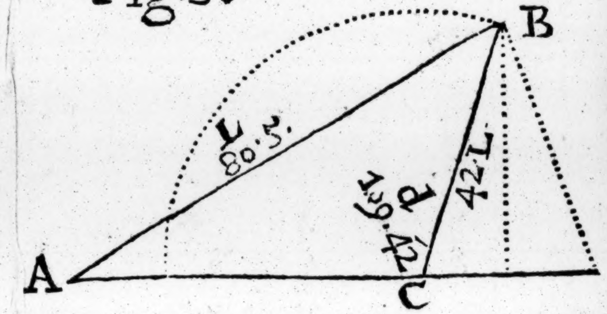


Fig. 30.



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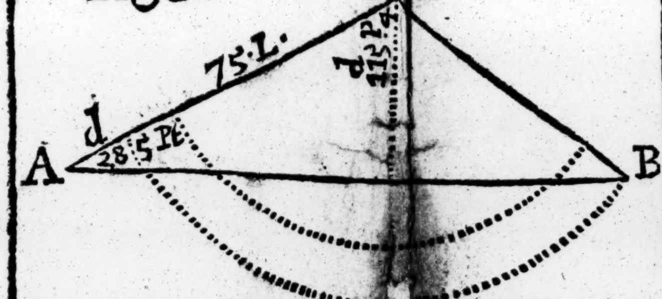


Fig. 32

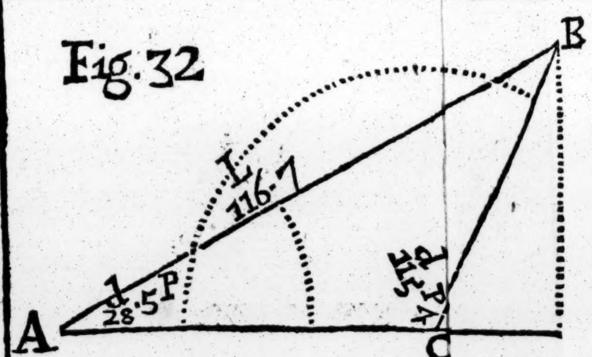
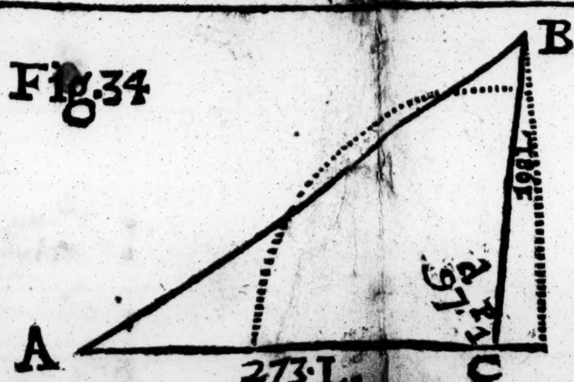


Fig. 33.

Fig. 34



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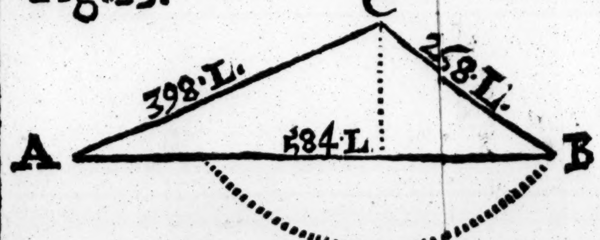

Fig. 36.

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His
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